# Improving the pedagogy of Capital Structure Theory: an Excel application 

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#### Abstract

This paper uses Excel to enhance the pedagogy of capital structure theory for corporate finance instructors and students. We provide a lesson plan that utilizes Excel spreadsheets and graphs to develop understanding of the theory. The theory is introduced in three scenarios that utilize Modigliani \& Miller's Propositions and "trade-off" theory.


Keywords: Capital structure, Modigliani \& Miller's Propositions, Excel spreadsheet.
JEL Classification: A22, A23, G00, G13, M20

## Introduction

This paper is motivated by students having difficulty understanding capital structure theory in senior undergraduate and MBA finance courses. This theory is demanding, typically commanding substantial coverage in intermediate corporate finance textbooks like Berk and DeMarz (2007), Brigham and Ehrhardt (2011), Brealey at al. (2008), and Ross et al. (2008). As such, finance instructors may find the task of imparting the theory pedagogically challenging.

Finance instructors would agree that the application of theory to specific problems that require calculations enhances student learning. In this regard, Excel applications are one of the most common methods used to explain elaborate financial theories. Cagle at al. (2010) explores the impact of spreadsheet applications in introductory finance courses and find positive effects in student exam scores. Consistent with this finding, there appears to be an increasing effort to incorporate Excel applications into finance textbooks, as exemplified by Holden (2005), Mayes and Shank (2010), Adair (2005), and Benninga (2010). Although these works provide extensive Excel applications for many corporate finance and investment topics, applications that focus specifically on capital structure are scant. This paper addresses the gap.

The proposed lesson plan starts in Section 2 with an elaboration of the value maximization equation that is central to capital structure theory by describing three scenarios that draw on the work of Modigliani and Miller $(1958,1963)$ and trade-off (or static) theory. In Section 3, Excel spreadsheets and graphs are used to apply the theory under each of these scenarios. A summary of the work is provided in Section 4.

## Capital Structure Theory

The earliest tenets of capital structure theory were developed by Modigliani and Miller $(1958,1963)$. The following value maximization equation is pedagogically central to the theory: $V_{0}=\sum_{t=1}^{n} \frac{F C F_{t}}{\left(1+r_{W A C C}\right)}$ where $F C F_{t}$ is the free cash flow at time t , and $r_{W A C C}$ is the average capital cost.

The negative correlation of firm value $\left(V_{0}\right)$ with average capital cost $\left(r_{\text {WACC }}\right)$ is a significant point. While M\&M's Proposition I focuses on $V_{0}$, their Proposition II focuses on $r_{\text {WACC }}$, with the particular values of $V_{0}$ and $r_{\text {WACC }}$ dependent on the scenario being contemplated. The scenarios are explained below and the associated equations are summarized in Table 1.

Scenario 1 assumes the absence of corporate taxes and the existence of perfect capital markets. M\&M Proposition I with no taxes implies that the values of levered $\left(V_{L}\right)$ and unlevered firms $\left(V_{U}\right)$ are equal, i.e. $V_{L}=V_{U}$. This condition is known as M\&M's "irrelevance hypothesis" and implies that financing through debt or equity does not affect firm value, and stays constant for different levels of leverage (debt). Similarly, M\&M's Proposition II shows that $r_{\text {WACC }}$ stays constant for different levels of debt as long as no corporate taxes and perfect capital markets are in place. Naturally, when $V_{L}$ and $r_{\text {WACC }}$ are constant at all debt levels, the firm's optimum debt ratio can be anywhere in the range of $0-100 \%$.

Scenario 2 relaxes the corporate tax assumption. To wit, it assumes the presence of corporate taxes and the existence of perfect capital markets. In the presence of corporate taxes, M\&M Proposition I implies that firm value increases with debt, i.e. $V_{L}=V_{U}+T_{C} D$, where $T_{C}$ is the corporate tax rate and $D$ refers to debt. The term $T_{C} D$ represents the additional firm value created by debt through a "tax shield." M\&M Proposition II in the presence of corporate taxes provides a similar outcome in that $r_{\text {WACC }}$ decreases as debt increases. The increasing $V_{L}$ and decreasing $r_{\text {WACC }}$ connote that the optimum debt level for the firm, in the presence of corporate taxes, is a $100 \%$ debt ratio.

Scenario 3 relaxes both the assumptions of corporate taxes and perfect capital markets. This realistic scenario is referred to as "trade-off" or "static" theory. The realistic scenario incorporates frictions that may arise due to the firm's high debt levels. When the M\&M propositions are modified to account for these frictions, the levered firm value takes a bell curved shape represented by $V_{L}=V_{U}+T_{C} D-f(D)$, where $f(D)$ is an additional function representing the reduction in firm value due to factors such as bankruptcy cost, increasing agency cost, underinvestment, turnover ratio among the employees, and the cost of financial distress. In this equation, the benefit provided by the tax shield $\left(T_{C} D\right)$ is reduced by the function $f(D)$, and the optimum total debt ratio lies somewhere between $0 \%$ and $100 \%$. A similar result is found for $r_{\text {WACC }}$, where an inverted bell curved shape optimizes at its minimum point.

## An Excel Application to Capital Structure Theory

Table 2 presents a lesson plan for instructors to use as an assignment for intermediate or advanced corporate finance students. Tables 3 and 4, which correspond to Scenario 1, generate $V_{U}, V_{L}, r_{E}$, and $r_{W A C C}$ values. Similarly, Tables 5 and 6 generate values for Scenario 2, and Tables 7 and 8 correspond to Scenario 3. Finally, Table 9 provides a graphical presentation of all three scenario outcomes from the application of the lesson plan. Instructors can use these graphs together with the Excel calculations, or independently, to summarize capital structure theory for students.

## Summary and Conclusions

After describing Modigliani and Miller's Propositions I and II, we develop a study plan for using Excel to improve the pedagogy of capital structure theory in intermediate and advanced corporate finance classes. The value and average cost of capital for a hypothetical firm XYZ is demonstrated in three scenarios with a step-by-step application of Excel spreadsheet and graphic capabilities.

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Table 1: The Summary of Equations for the Three Scenarios of Capital Structure Theory
Scenario 1: Assume no taxes and perfect capital markets

| FIRM VALUE: M\&M PROPOSITION I | $V_{L}=V_{U}=\frac{E B I T}{r_{0}}$ |
| :--- | :--- |
|  | $r_{E}=r_{0}+\left(r_{0}-r_{D}\right)\left(\frac{D}{E}\right)$ |
| COST OF CAPITAL: M\&M PROPOSITION | $r_{D}=a$ where a is a constant percent rate. |
| II | $r_{W A C C}=\left(\frac{E}{E+D}\right) r_{E}+\left(\frac{D}{E+D}\right) r_{D}$ |

Scenario 2: Corporate taxes are introduced into perfect capital markets

| FIRM VALUE: M\&M PROPOSITION I |  |  | $\begin{gathered} V_{L}=V_{U}+T_{C} D \\ \\ E B I T\left(1-T_{C}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: |
|  |  |  | $r_{0}$ |
| $\begin{array}{lc} \text { COST OF } \\ \text { PROPOSITION II } \end{array}$ | CAPITAL: | M\&M | $r_{E}=r_{0}+\left(1-T_{C}\right)\left(r_{0}-r_{D}\right)\left(\frac{D}{E}\right)$ <br> $r_{D}=\left(1-T_{C}\right) a$ where a is a constant percent rate, and $r_{D}$ is after tax debt rate. $r_{W A C C}=\left(\frac{E}{E+D}\right) r_{E}+\left(\frac{D}{E+D}\right)\left(1-T_{C}\right) r_{D}$ |

Scenario 3: With all types of frictions including taxes and therefore, there is no perfect capital market assumption

| FIRM VALUE: REALISTIC CASE KNOWN AS EITHER "TRADE OFF" THEORY OR "STATIC" THEORY | $\begin{aligned} & V_{L}=V_{U}+T_{C} D-f(D) \\ & V_{L}=\frac{E B I T\left(1-T_{C}\right)}{r_{0}}+T_{C} D-f(D) \end{aligned}$ |
| :---: | :---: |
| COST OF CAPITAL: REALISTIC CASE KNOWN AS EITHER "TRADE OFF" THEORY OR "STATIC" THEORY | $r_{E}=r_{0}+\left(1-T_{C}\right)\left(r_{0}-r_{D}\right)\left(\frac{D}{E}\right)^{2}$ <br> $r_{D}^{* *}=\left(1-T_{C}\right) a+b\left(\frac{D}{E}\right)^{2} \quad$ where $\quad a \quad$ and constant percent rates, and $r_{D}$ is after rate. $r_{W A C C}=\left(\frac{E}{E+D}\right) r_{E}+\left(\frac{D}{E+D}\right)\left(1-T_{C}\right) r_{D}$ |

Table 2: A Lesson Plan for Capital Structure Theory with Excel
Scenario 1: Finding $V_{U}, V_{L}, r_{E}, r_{\text {WACC }}$ with M\&M Propositions I and II and No Corporate Taxes
Copy and paste the following information into an Excel worksheet and name the tab as "Section1":

| $\mathbf{r 0}$ | rD | $\mathbf{E B I T}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{D}+\mathbf{E}$ | $\mathbf{D} / \mathbf{E}$ | $\mathbf{V U}$ | $\mathbf{V L}$ | $\mathbf{r E}$ | rWACC |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $20 \%$ | $5 \%$ | 20 | 0 | 100 | 100 | 0.0 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 10 | 100 | 110 | 0.1 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 20 | 100 | 120 | 0.2 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 30 | 100 | 130 | 0.3 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 40 | 100 | 140 | 0.4 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 50 | 100 | 150 | 0.5 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 60 | 100 | 160 | 0.6 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 70 | 100 | 170 | 0.7 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 80 | 100 | 180 | 0.8 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 90 | 100 | 190 | 0.9 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 100 | 100 | 200 | 1.0 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 110 | 100 | 210 | 1.1 |  |  |  |  |
| $20 \%$ | $5 \%$ | 20 | 120 | 100 | 220 | 1.2 |  |  |  |  |

Assume that Corporation XYZ has a required return on equity of $20 \%$ when the firm is all equity (no debt), and it can borrow loans at a constant rate of $5 \%$. Additionally, assume that XYZ has a perpetual EBIT of $\$ 20$.
a. Find unlevered ( $V_{U}$ ) and levered ( $V_{L}$ ) firm values based on the equation:
$V_{L}=V_{U}=\frac{E B I T}{r_{0}}$.
b. Similarly find required return on equity $\left(r_{E}\right)$ by using $r_{E}=r_{0}+\left(r_{0}-r_{D}\right)\left(\frac{D}{E}\right)$, and average capital cost $\left(r_{W A C C}\right)$ by using $r_{W A C C}=\left(\frac{E}{E+D}\right) r_{E}+\left(\frac{D}{E+D}\right) r_{D}$.
c. Graph $V_{L}$ for all debt levels.
d. Graph $r_{E}, r_{D}$, and $r_{\text {WACC }}$ for all (D/E) ratio ranges.

Scenario 2: Finding $V_{U}, V_{L}, r_{E}, r_{D}$, and $r_{\text {WACC }}$ with M\&M Propositions I and II with Corporate Taxes
Copy and paste the table from Section 1 in another worksheet and name the tab as
"Section2" assuming a corporate tax rate of $40 \%$ applies to Corporation XYZ. Also, change the values of E to be $\$ 60$ for the start for every D levels (see the note below):
a. Find unlevered ( $V_{U}$ ) and levered ( $V_{L}$ ) firm values based on the equation:
$V_{L}=V_{U}+T_{C} D$ and
$V_{L}=\frac{E B I T\left(1-T_{C}\right)}{r_{0}}+T_{C} D$
b. Similarly find required return on equity $\left(r_{E}\right)$ by using $r_{E}=r_{0}+\left(1-T_{C}\right)\left(r_{0}-r_{D}\right)\left(\frac{D}{E}\right)$, the cost of debt $\left(r_{D}\right)$ by using $r_{D}^{*}=\left(1-T_{C}\right) a$, and average capital cost $\left(r_{W A C C}\right)$ by using $r_{W A C C}=\left(\frac{E}{E+D}\right) r_{E}+\left(\frac{D}{E+D}\right) r_{D}^{* *}$.
c. Graph $V_{L}$ for all debt levels.
d. Graph $r_{E}, r_{D}$, and $r_{\text {WACC }}$ for all (D/E) ratio ranges.

Section 3: Finding $V_{U}, V_{L}, r_{E}, r_{W A C C}$ with M\&M Propositions I and II with Corporate Taxes and all frictions
Copy and paste the table from Section 1 in another worksheet and name the tab as "Section3" using the same assumptions as in Section 2:
a. Find unlevered ( $V_{U}$ ) and levered $\left(V_{L}\right)$ firm values based on the equations:
$V_{L}=V_{U}+T_{C} D-f(D)$ and $V_{L}=\frac{E B I T\left(1-T_{C}\right)}{r_{0}}+T_{C} D-f(D)$. Assume a function value
for $f(D)=0.01(T)(D)^{2}$ as a start.
b. Similarly find the required return on equity $\left(r_{E}\right)$ by using,
$r_{E}=r_{0}+\left(1-T_{C}\right)\left(r_{0}-r_{D}\right)\left(\frac{D}{E}\right)^{2}$, the cost of debt $\left(r_{D}\right)$ by using $r_{D}^{* *}=\left(1-T_{C}\right) a+b\left(\frac{D}{E}\right)^{2}$, and the average capital cost $\left(r_{W A C C}\right)$ by using $r_{W A C C}=\left(\frac{E}{E+D}\right) r_{E}+\left(\frac{D}{E+D}\right) r_{D}^{* *}$. Assume a function value of $r_{D}^{* *}=(1-.40)(.05)+(.05)\left(\frac{D}{E}\right)^{2}$ as a start.
c. Graph $V_{L}$ for all debt levels.
d. Graph $r_{E}, r_{D}$, and $r_{W A C C}$ for all (D/E) ratio ranges.

Note: In Section 1, E is $\$ 100$ with no corporate taxes assumption and $\mathrm{D}=\$ 0$ as a start. Thus, the levered and unlevered firm values are equal; $\mathrm{VL}=\mathrm{VU}=\mathrm{EBIT} / \mathrm{r} 0=20 / .20=\$ 100$. In Sections 2 and 3 with corporate taxes assumption and when $\mathrm{D}=\$ 0$, E becomes $\$ 60$ since $\mathrm{VL}=[(1-\mathrm{Tc}) \mathrm{EBIT}] / \mathrm{r} 0+\mathrm{TcD}$ and $\mathrm{VL}=[(1-.40) 20] / .20+.40(0)=\$ 60$ due to $V=E+D$.

Table 3: Firm Value and Cost of capital with M\&M Propositions I and II with No Corporate Taxes (Scenario 1)

|  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{I}$ | $\mathbf{J}$ | $\mathbf{K}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | $\mathbf{r 0}$ | $\mathbf{r D}$ | $\mathbf{E B I T}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{D}+\mathbf{E}$ | $\mathbf{D} / \mathbf{E}$ | $\mathbf{V U}$ | $\mathbf{V L}$ | $\mathbf{r E}$ | $\mathbf{\text { rWACC }}$ |
| $\mathbf{2}$ | $20 \%$ | $5 \%$ | 20 | 0 | 100 | 100 | 0.0 | 100 | 100 | $20 \%$ | $20 \%$ |
| $\mathbf{3}$ | $20 \%$ | $5 \%$ | 20 | 10 | 100 | 110 | 0.1 | 100 | 100 | $22 \%$ | $20 \%$ |
| $\mathbf{4}$ | $20 \%$ | $5 \%$ | 20 | 20 | 100 | 120 | 0.2 | 100 | 100 | $23 \%$ | $20 \%$ |
| $\mathbf{5}$ | $20 \%$ | $5 \%$ | 20 | 30 | 100 | 130 | 0.3 | 100 | 100 | $25 \%$ | $20 \%$ |
| $\mathbf{6}$ | $20 \%$ | $5 \%$ | 20 | 40 | 100 | 140 | 0.4 | 100 | 100 | $26 \%$ | $20 \%$ |
| $\mathbf{7}$ | $20 \%$ | $5 \%$ | 20 | 50 | 100 | 150 | 0.5 | 100 | 100 | $28 \%$ | $20 \%$ |
| $\mathbf{8}$ | $20 \%$ | $5 \%$ | 20 | 60 | 100 | 160 | 0.6 | 100 | 100 | $29 \%$ | $20 \%$ |
| $\mathbf{9}$ | $20 \%$ | $5 \%$ | 20 | 70 | 100 | 170 | 0.7 | 100 | 100 | $31 \%$ | $20 \%$ |
| $\mathbf{1 0}$ | $20 \%$ | $5 \%$ | 20 | 80 | 100 | 180 | 0.8 | 100 | 100 | $32 \%$ | $20 \%$ |
| $\mathbf{1 1}$ | $20 \%$ | $5 \%$ | 20 | 90 | 100 | 190 | 0.9 | 100 | 100 | $34 \%$ | $20 \%$ |
| $\mathbf{1 2}$ | $20 \%$ | $5 \%$ | 20 | 100 | 100 | 200 | 1.0 | 100 | 100 | $35 \%$ | $20 \%$ |
| $\mathbf{1 3}$ | $20 \%$ | $5 \%$ | 20 | 110 | 100 | 210 | 1.1 | 100 | 100 | $37 \%$ | $20 \%$ |
| $\mathbf{1 4}$ | $20 \%$ | $5 \%$ | 20 | 120 | 100 | 220 | 1.2 | 100 | 100 | $38 \%$ | $20 \%$ |

Note:
We assume that the firm XYZ has a required return on equity (r0) is $20 \%$ when the firm is all equity (no debt), and the firm has a constant borrowing rate (rD) of 5\%. Additionally, we assume that XYZ has a perpetual EBIT of \$20.

Table 4: Underlying Formulas for the Firm Value and Cost of capital with M\&M Propositions I and II with No Corporate Taxes (Scenario 1)

|  | A | B | C | D | E | F | G | H | I | J | K |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | r0 | rD | EBIT | D | E | D+E | D/E | VU | VL | rE | rWACC |
| 2 | 0.2 | 0.05 | 20 | 0 | 100 | =D2+E2 | =D2/E2 | $=\mathrm{C} 2 / \mathrm{A} 2$ | =C2/K2 | =A2+G2*(A2-B2) | $=(\mathrm{D} 2 / \mathrm{F} 2) * \mathrm{~B} 2+(\mathrm{E} 2 / \mathrm{F} 2) * \mathrm{~J} 2$ |
| 3 | 0.2 | 0.05 | 20 | =D2+10 | 100 | =D3+E3 | =D3/E3 | $=\mathrm{C} 3 / \mathrm{A} 3$ | =C3/K3 | =A3+G3*(A3-B3) | $=(\mathrm{D} 3 / \mathrm{F} 3) * \mathrm{~B} 3+(\mathrm{E} 3 / \mathrm{F} 3) * \mathrm{~J} 3$ |
| 4 | 0.2 | 0.05 | 20 | =D3+10 | 100 | =D4+E4 | =D4/E4 | = $\mathrm{C} 4 / \mathrm{A} 4$ | =C4/K4 | =A4+G4*(A4-B4) | $=(\mathrm{D} 4 / \mathrm{F} 4) * \mathrm{~B} 4+(\mathrm{E} 4 / \mathrm{F} 4) * \mathrm{~J} 4$ |
| 5 | 0.2 | 0.05 | 20 | =D4+10 | 100 | =D5+E5 | =D5/E5 | = C5/A5 | =C5/K5 | =A5+G5*(A5-B5) | $=($ D5/F5 $) *$ B5 + (E5/F5)*J5 |
| 6 | 0.2 | 0.05 | 20 | =D5+10 | 100 | =D6+E6 | =D6/E6 | =C6/A6 | =C6/K6 | =A6+G6*(A6-B6) | $=(\mathrm{D} 6 / \mathrm{F} 6) * \mathrm{~B} 6+(\mathrm{E} 6 / \mathrm{F} 6) * \mathrm{~J} 6$ |
| 7 | 0.2 | 0.05 | 20 | =D6+10 | 100 | = $\mathrm{D} 7+\mathrm{E} 7$ | =D7/E7 | $=\mathrm{C} 7 / \mathrm{A} 7$ | =C7/K7 | =A7+G7*(A7-B7) | $=(\mathrm{D} 7 / \mathrm{F} 7) * \mathrm{~B} 7+(\mathrm{E} 7 / \mathrm{F} 7) * \mathrm{~J} 7$ |
| 8 | 0.2 | 0.05 | 20 | = $77+10$ | 100 | =D8+E8 | =D8/E8 | = $\mathrm{C} 8 / \mathrm{A} 8$ | =C8/K8 | =A8+G8*(A8-B8) | $=(\mathrm{D} 8 / \mathrm{F} 8) * \mathrm{~B} 8+(\mathrm{E} 8 / \mathrm{F} 8) * \mathrm{~J} 8$ |
| 9 | 0.2 | 0.05 | 20 | =D8+10 | 100 | =D9+E9 | =D9/E9 | =C9/A9 | =C9/K9 | =A9+G9*(A9-B9) | $=(\mathrm{D} 9 / \mathrm{F9}) * \mathrm{B9} 9+(\mathrm{E} 9 / \mathrm{F9} 9 *$ J9 |
| 10 | 0.2 | 0.05 | 20 | =D9+10 | 100 | =D10+E10 | =D10/E10 | = C10/A10 | =C10/K10 | =A10+G10*(A10-B10) | $=(\mathrm{D} 10 / \mathrm{F} 10) * \mathrm{~B} 10+(\mathrm{E} 10 / \mathrm{F} 10) * \mathrm{~J} 10$ |
| 11 | 0.2 | 0.05 | 20 | =D10+10 | 100 | =D11+E11 | =D11/E11 | =C11/A11 | =C11/K11 | $=\mathrm{A} 11+\mathrm{G} 11{ }^{*}(\mathrm{~A} 11-\mathrm{B} 11)$ | $=(\mathrm{D} 11 / \mathrm{F} 11) * \mathrm{~B} 11+(\mathrm{E} 11 / \mathrm{F} 11) * \mathrm{~J} 11$ |
| 12 | 0.2 | 0.05 | 20 | =D11+10 | 100 | =D12+E12 | =D12/E12 | =C12/A12 | =C12/K12 | $=\mathrm{A} 12+\mathrm{G} 12 *$ (A12-B12) | $=(\mathrm{D} 12 / \mathrm{F} 12) * \mathrm{~B} 12+(\mathrm{E} 12 / \mathrm{F} 12) * \mathrm{~J} 12$ |
| 13 | 0.2 | 0.05 | 20 | =D12+10 | 100 | =D13+E13 | =D13/E13 | =C13/A13 | =C13/K13 | =A13+G13*(A13-B13) | $=(\mathrm{D} 13 / \mathrm{F} 13) * \mathrm{~B} 13+(\mathrm{E} 13 / \mathrm{F} 13) * \mathrm{~J} 13$ |
| 14 | 0.2 | 0.05 | 20 | =D13+10 | 100 | =D14+E14 | =D14/E14 | =C14/A14 | =C14/K14 | =A14+G14*(A14-B14) | $=($ D14/F14)*B14+(E14/F14)*J14 |

Note:
We assume that the firm XYZ has a required return on equity (r0) is $20 \%$ when the firm is all equity (no debt), and the firm has a constant borrowing rate (rD) of $5 \%$. Additionally, we assume that XYZ has a perpetual EBIT of $\$ 20$.

Table 5: Firm Value and Cost of capital with M\&M Propositions I and II with Corporate Taxes (Scenario 2)

|  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{I}$ | $\mathbf{J}$ | $\mathbf{K}$ | $\mathbf{L}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | $\mathbf{r 0}$ | $\mathbf{r D}$ | $\mathbf{E B I T}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{D}+\mathbf{E}$ | $\mathbf{D} / \mathbf{E}$ | $\mathbf{T}$ | $\mathbf{V} \mathbf{M}$ | $\mathbf{V L}$ | $\mathbf{r E}$ | $\mathbf{( 1 - T ) \mathbf { r D }}$ | $\mathbf{\text { rWACC }}$ |
| $\mathbf{2}$ | $20 \%$ | $5 \%$ | 20 | 0 | 60 | 60 | 0.0 | 0.4 | 60 | 60 | $20 \%$ | $3 \%$ | $20 \%$ |
| $\mathbf{3}$ | $20 \%$ | $5 \%$ | 20 | 10 | 60 | 70 | 0.2 | 0.4 | 60 | 64 | $22 \%$ | $3 \%$ | $19 \%$ |
| $\mathbf{4}$ | $20 \%$ | $5 \%$ | 20 | 20 | 60 | 80 | 0.3 | 0.4 | 60 | 68 | $23 \%$ | $3 \%$ | $18 \%$ |
| $\mathbf{5}$ | $20 \%$ | $5 \%$ | 20 | 30 | 60 | 90 | 0.5 | 0.4 | 60 | 72 | $25 \%$ | $3 \%$ | $17 \%$ |
| $\mathbf{6}$ | $20 \%$ | $5 \%$ | 20 | 40 | 60 | 100 | 0.7 | 0.4 | 60 | 76 | $26 \%$ | $3 \%$ | $17 \%$ |
| $\mathbf{7}$ | $20 \%$ | $5 \%$ | 20 | 50 | 60 | 110 | 0.8 | 0.4 | 60 | 80 | $28 \%$ | $3 \%$ | $16 \%$ |
| $\mathbf{8}$ | $20 \%$ | $5 \%$ | 20 | 60 | 60 | 120 | 1.0 | 0.4 | 60 | 84 | $29 \%$ | $3 \%$ | $16 \%$ |
| $\mathbf{9}$ | $20 \%$ | $5 \%$ | 20 | 70 | 60 | 130 | 1.2 | 0.4 | 60 | 88 | $31 \%$ | $3 \%$ | $16 \%$ |
| $\mathbf{1 0}$ | $20 \%$ | $5 \%$ | 20 | 80 | 60 | 140 | 1.3 | 0.4 | 60 | 92 | $32 \%$ | $3 \%$ | $15 \%$ |
| $\mathbf{1 1}$ | $20 \%$ | $5 \%$ | 20 | 90 | 60 | 150 | 1.5 | 0.4 | 60 | 96 | $34 \%$ | $3 \%$ | $15 \%$ |
| $\mathbf{1 2}$ | $20 \%$ | $5 \%$ | 20 | 100 | 60 | 160 | 1.7 | 0.4 | 60 | 100 | $35 \%$ | $3 \%$ | $15 \%$ |
| $\mathbf{1 3}$ | $20 \%$ | $5 \%$ | 20 | 110 | 60 | 170 | 1.8 | 0.4 | 60 | 104 | $37 \%$ | $3 \%$ | $15 \%$ |
| $\mathbf{1 4}$ | $20 \%$ | $5 \%$ | 20 | 120 | 60 | 180 | 2.0 | 0.4 | 60 | 108 | $38 \%$ | $3 \%$ | $15 \%$ |

Notes:

1. We assume that the firm $X Y Z$ has a required return on equity (r0) is $20 \%$ when the firm is all equity (no debt), and the firm has a constant borrowing rate (rD) of $5 \%$. Additionally, we assume that XYZ has a perpetual EBIT of $\$ 20$.
2. The tax rate $(\mathrm{T})$ is $40 \%$ and (1-T)rD after tax is added to produce after tax cost of debt.
3. In Section 1, E is $\$ 100$ with no corporate taxes assumption and $\mathrm{D}=\$ 0$ as a start. Thus, the levered and unlevered firm values are equal; $\mathrm{VL}=\mathrm{VU}=\mathrm{EBIT} / \mathrm{r} 0=20 / .20=\$ 100$. In Sections 2 and 3 with corporate taxes assumption and when $\mathrm{D}=\$ 0$, E becomes $\$ 60$ since $\mathrm{VL}=[(1-\mathrm{Tc}) \mathrm{EBIT}] / \mathrm{r} 0+\mathrm{TcD}$ and $\mathrm{VL}=[(1-.40) 20] / .20+.40(0)=\$ 60$ due to $\mathrm{V}=\mathrm{E}+\mathrm{D}$.

Table 6: Underlying Formulas for the Firm Value and Cost of capital with M\&M Propositions I and II with Corporate Taxes (Scenario 2)

|  | I | J | K | L | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | VU | VL | rE | (1-T)*rD | rWACC |
| 2 | $=\mathrm{C} 2 *(1-\mathrm{H} 2) / \mathrm{A} 2$ | $=(\mathrm{C} 2 *(1-\mathrm{H} 2)$ )/ $22+\mathrm{H} 2 * \mathrm{D} 2$ | $=\mathrm{A} 2+(1-\mathrm{H} 2) *(\mathrm{~A} 2-\mathrm{B} 2) * \mathrm{G} 2$ | $=(1-\mathrm{H} 2) * \mathrm{~B} 2$ | $=(\mathrm{E} 2 / \mathrm{F} 2) * \mathrm{~K} 2+(\mathrm{D} 2 / \mathrm{F} 2) * \mathrm{~L} 2$ |
| 3 | $=\mathrm{C} 3 *(1-\mathrm{H}) / \mathrm{A} 3$ | $=(\mathrm{C} 3 *(1-\mathrm{H} 3)) / \mathrm{A} 3+\mathrm{H} 3 * \mathrm{D} 3$ | $=\mathrm{A} 3+(1-\mathrm{H} 3) *$ ( $\mathrm{A} 3-\mathrm{B} 3) * \mathrm{G} 3$ | $=(1-\mathrm{H} 3) *$ B3 | $=(\mathrm{E} 3 / \mathrm{F} 3) * \mathrm{~K} 3+(\mathrm{D} 3 / \mathrm{F} 3) * \mathrm{~L} 3$ |
| 4 | $=\mathrm{C} 4 *(1-\mathrm{H} 4) / \mathrm{A} 4$ | $=(\mathrm{C} 4 *(1-\mathrm{H} 4)$ )/ $44+\mathrm{H} 4 * \mathrm{D} 4$ | $=\mathrm{A} 4+(1-\mathrm{H} 4) *(\mathrm{~A} 4-\mathrm{B} 4) * \mathrm{G} 4$ | $=(1-\mathrm{H} 4) * \mathrm{~B} 4$ | $=(\mathrm{E} 4 / \mathrm{F} 4) * \mathrm{~K} 4+$ (D4/F4)*L4 |
| 5 | = $\mathrm{C} 5 *(1-\mathrm{H} 5) / \mathrm{A} 5$ | $=(\mathrm{C} 5 *(1-\mathrm{H} 5)$ )/A5+H5*D5 | $=\mathrm{A} 5+(1-\mathrm{H} 5) *$ (A5-B5)*G5 | $=(1-\mathrm{H} 5) * \mathrm{~B} 5$ | $=(\mathrm{E} 5 / \mathrm{F}) *$ K $5+$ (D5/F5)*L5 |
| 6 | $=\mathrm{C} 6 *(1-\mathrm{H} 6) / \mathrm{A} 6$ | $=(\mathrm{C} 6 *(1-\mathrm{H} 6)$ )/A6+H6*D6 | $=\mathrm{A} 6+(1-\mathrm{H} 6) *$ (A6-B6)*G6 | $=(1-\mathrm{H} 6) * \mathrm{~B} 6$ | $=(\mathrm{E} 6 \mathrm{~F} 6) * \mathrm{~K} 6+$ (D6/F6)*L6 |
| 7 | $=\mathrm{C} 7 *(1-\mathrm{H} 7$ )/A7 | $=(\mathrm{C} 7 *(1-\mathrm{H} 7)$ )/ $\mathrm{A} 7+\mathrm{H} 7 * \mathrm{D} 7$ | $=\mathrm{A} 7+(1-\mathrm{H} 7) *(\mathrm{~A} 7-\mathrm{B} 7) * \mathrm{G} 7$ | $=(1-\mathrm{H} 7) * \mathrm{~B} 7$ | $=(\mathrm{E} 7 \mathrm{~F} 7) * \mathrm{~K} 7+(\mathrm{D} 7 / \mathrm{F} 7) * \mathrm{~L} 7$ |
| 8 | $=\mathrm{C} 8 *(1-\mathrm{H}) / \mathrm{A} 8$ | $=(\mathrm{C} 8 *(1-\mathrm{H} 8)$ )/A8+H8*D8 | $=\mathrm{A} 8+(1-\mathrm{H} 8) *$ (A8-B8)*G8 | $=(1-\mathrm{H} 8) * \mathrm{~B} 8$ | $=(\mathrm{E} 8 / \mathrm{F} 8) * \mathrm{~K} 8+(\mathrm{D} 8 / \mathrm{F} 8) * \mathrm{~L} 8$ |
| 9 | = $\mathrm{C} 9 *(1-\mathrm{H} 9) / \mathrm{A} 9$ | $=(\mathrm{C} 9 *(1-\mathrm{H} 9)$ )/A9+H9*D9 | =A9+(1-H9)*(A9-B9)*G9 | = $11-\mathrm{H} 9$ )*B9 | =(E9/F9)*K9+(D9/F9)*L9 |
| 10 | $=\mathrm{Cl} 10 *(1-\mathrm{H} 10) / \mathrm{Al0}$ | $=(\mathrm{Cl0}$ ( $1-\mathrm{H} 10)$ )/A10+H10*D10 | $=\mathrm{Al0}+(1-\mathrm{H} 10) *(\mathrm{Al0-B10}) * \mathrm{G} 10$ | $=(1-\mathrm{H} 10) * \mathrm{~B} 10$ | $=(\mathrm{E} 10 / \mathrm{F} 10) * \mathrm{~K} 10+$ (D10/F10)*L10 |
| 11 | $=\mathrm{Cl1} *(1-\mathrm{Hl1}) / \mathrm{Al1}$ | $=(\mathrm{Cl1} *(1-\mathrm{Hl1})) / \mathrm{Al1}+\mathrm{Hl1} * \mathrm{D} 11$ | $=\mathrm{Al1}+(1-\mathrm{Hl1}) *(\mathrm{Al1-B11)}$ * G 11 | $=(1-\mathrm{H} 11) * \mathrm{Bl1}$ | $=($ E11/F11)*K11+(D11/F11)*L11 |
| 12 | $=\mathrm{Cl2} *(1-\mathrm{H} 12) / \mathrm{Al2}$ | $=(\mathrm{Cl2} 2(1-\mathrm{Hl2}) / \mathrm{/A} 12+\mathrm{H} 12 * \mathrm{D} 12$ | $=\mathrm{Al2}+(1-\mathrm{H} 12) *(\mathrm{Al2-B12)}$ * G 12 | $=(1-\mathrm{H} 12) * \mathrm{~B} 12$ | $=($ E12/F12)*K12+(D12/F12)*L12 |
| 13 | $=\mathrm{Cl} 3 *(1-\mathrm{H} 13) / \mathrm{Al3}$ | $=(\mathrm{Cl3} *(1-\mathrm{Hl} 3) / \mathrm{/A13}+\mathrm{H} 13 * \mathrm{D} 13$ | $=\mathrm{Al3}+(1-\mathrm{H} 13) *(\mathrm{Al3}-\mathrm{B} 13) * \mathrm{G} 13$ | $=(1-\mathrm{H} 13) * \mathrm{Bl} 3$ | $=(\mathrm{E} 13 / \mathrm{F} 13) * \mathrm{~K} 13+$ (D13/F13)*L13 |
| 14 | $=\mathrm{C} 14 *(1-\mathrm{H} 14) / \mathrm{Al4}$ | $=(\mathrm{Cl4*}(1-\mathrm{H} 14)) / \mathrm{A} 14+\mathrm{H} 14 * \mathrm{D} 14$ | $=\mathrm{Al4}+(1-\mathrm{H} 14) *(\mathrm{~A} 14-\mathrm{B} 14) * \mathrm{G} 14$ | $=(1-\mathrm{H} 14) *$ B14 | $=($ E14/F14)*K14+(D14/F14)*L14 |

Notes:

1. We assume that the firm $X Y Z$ has a required return on equity (r0) is $20 \%$ when the firm is all equity (no debt), and the firm has a constant borrowing rate (rD) of 5\%. Additionally, we assume that XYZ has a perpetual EBIT of \$20.
2. The tax rate ( T ) is $40 \%$ and (1-T)rD after tax is added to produce after tax cost of debt.
3. In Section 1, E is $\$ 100$ with no corporate taxes assumption and $\mathrm{D}=\$ 0$ as a start. Thus, the levered and unlevered firm values are equal; $\mathrm{VL}=\mathrm{VU}=\mathrm{EBIT} / \mathrm{r} 0=20 / .20=\$ 100$. In Sections 2 and 3 with corporate taxes assumption and when $\mathrm{D}=\$ 0$, E becomes $\$ 60$ since $\mathrm{VL}=[(1-\mathrm{Tc}) \mathrm{EBIT}] / \mathrm{r} 0+\mathrm{TcD}$ and $\mathrm{VL}=[(1-.40) 20] / .20+.40(0)=\$ 60$ due to $\mathrm{V}=\mathrm{E}+\mathrm{D}$.

Table 7: Firm Value and Cost of capital with Realistic Case of Trade-off or Static Theory (Scenario 3)

|  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{I}$ | $\mathbf{J}$ | $\mathbf{K}$ | $\mathbf{L}$ | $\mathbf{M}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | $\mathbf{r 0}$ | $\mathbf{r D}$ | $\mathbf{E B I T}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{D}+\mathbf{E}$ | $\mathbf{D} / \mathbf{E}$ | $\mathbf{T}$ | $\mathbf{V} \mathbf{V}$ | $\mathbf{V L}$ | $\mathbf{r E}$ | $\mathbf{r D}{ }^{*}$ | $\mathbf{r W A C C}$ |
| $\mathbf{2}$ | $20 \%$ | $5 \%$ | 20 | 0 | 60 | 60 | 0.0 | 0.4 | 60 | 60.0 | $20 \%$ | $3.00 \%$ | $20.00 \%$ |
| $\mathbf{3}$ | $20 \%$ | $5 \%$ | 20 | 10 | 60 | 70 | 0.2 | 0.4 | 60 | 63.6 | $20 \%$ | $3.14 \%$ | $17.63 \%$ |
| $\mathbf{4}$ | $20 \%$ | $5 \%$ | 20 | 20 | 60 | 80 | 0.3 | 0.4 | 60 | 66.4 | $21 \%$ | $3.56 \%$ | $16.28 \%$ |
| $\mathbf{5}$ | $20 \%$ | $5 \%$ | 20 | 30 | 60 | 90 | 0.5 | 0.4 | 60 | 68.4 | $22 \%$ | $4.25 \%$ | $15.68 \%$ |
| $\mathbf{6}$ | $20 \%$ | $5 \%$ | 20 | 40 | 60 | 100 | 0.7 | 0.4 | 60 | 69.6 | $24 \%$ | $5.22 \%$ | $15.65 \%$ |
| $\mathbf{7}$ | $20 \%$ | $5 \%$ | 20 | 50 | 60 | 110 | 0.8 | 0.4 | 60 | 70.0 | $26 \%$ | $6.47 \%$ | $16.08 \%$ |
| $\mathbf{8}$ | $20 \%$ | $5 \%$ | 20 | 60 | 60 | 120 | 1.0 | 0.4 | 60 | 69.6 | $29 \%$ | $8.00 \%$ | $16.90 \%$ |
| $\mathbf{9}$ | $20 \%$ | $5 \%$ | 20 | 70 | 60 | 130 | 1.2 | 0.4 | 60 | 68.4 | $32 \%$ | $9.81 \%$ | $18.05 \%$ |
| $\mathbf{1 0}$ | $20 \%$ | $5 \%$ | 20 | 80 | 60 | 140 | 1.3 | 0.4 | 60 | 66.4 | $36 \%$ | $11.89 \%$ | $19.50 \%$ |
| $\mathbf{1 1}$ | $20 \%$ | $5 \%$ | 20 | 90 | 60 | 150 | 1.5 | 0.4 | 60 | 63.6 | $40 \%$ | $14.25 \%$ | $21.23 \%$ |
| $\mathbf{1 2}$ | $20 \%$ | $5 \%$ | 20 | 100 | 60 | 160 | 1.7 | 0.4 | 60 | 60.0 | $45 \%$ | $16.89 \%$ | $23.21 \%$ |
| $\mathbf{1 3}$ | $20 \%$ | $5 \%$ | 20 | 110 | 60 | 170 | 1.8 | 0.4 | 60 | 55.6 | $50 \%$ | $19.81 \%$ | $25.42 \%$ |
| $\mathbf{1 4}$ | $20 \%$ | $5 \%$ | 20 | 120 | 60 | 180 | 2.0 | 0.4 | 60 | 50.4 | $56 \%$ | $23.00 \%$ | $27.87 \%$ |

Notes:

1. We assume that the firm $X Y Z$ has a required return on equity (r0) is $20 \%$ when the firm is all equity (no debt), and the firm has a constant borrowing rate (rD) of $5 \%$. Additionally, we assume that XYZ has a perpetual EBIT of $\$ 20$.
2. The levered firm value is generated by $V_{L}=V_{U}+T_{C} D-f(D)$ and
$V_{L}=\frac{E B I T\left(1-T_{C}\right)}{r_{0}}+T_{C} D-f(D)$ where

$$
f(D)=0.01(T)(D)^{2} .
$$

3. To generate rE , we used $r_{E}=r_{0}+\left(1-T_{C}\right)\left(r_{0}-r_{D}\right)\left(\frac{D}{E}\right)^{2}$.
4. The tax rate (T) is $40 \%$ and the cost of debt after tax is $r_{D}^{* *}=\left(1-T_{C}\right) a+b\left(\frac{D}{E}\right)^{2}$ and applied as

$$
r_{D}^{* *}=(1-.40)(.05)+(.05)\left(\frac{D}{E}\right)^{2}
$$

5. In Section 1, E is $\$ 100$ with no corporate taxes assumption and $\mathrm{D}=\$ 0$ as a start. Thus, the levered and unlevered firm values are equal; $\mathrm{V}_{\mathrm{L}}=\mathrm{VU}=\mathrm{EBIT} / \mathrm{r}_{0}=20 / .20=\$ 100$. In Sections 2 and 3 with corporate taxes assumption and when $\mathrm{D}=\$ 0$, E becomes $\$ 60$ since $\mathrm{V}_{\mathrm{L}}=[(1-\mathrm{Tc}) \mathrm{EBIT}] / \mathrm{r}_{0}+\mathrm{TcD}$ and $\mathrm{V}_{\mathrm{L}}=[(1-.40) 20] / .20+.40(0)=\$ 60$ due to $\mathrm{V}=\mathrm{E}+\mathrm{D}$.

Table 8: Underlying Formulas for the Firm Value and Cost of capital in the Realistic Case of Trade-off or Static Theory (Scenario 3)

|  | I | J | K | L | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | vu | VL | rE | rD* | rWACC |
| 2 | $=\mathrm{C} 2 *(1-\mathrm{H} 2) / \mathrm{A} 2$ | $=(\mathrm{C} 2 *(1-\mathrm{H} 2) / \mathrm{A} 2+\mathrm{H} 2 * \mathrm{D} 2-0.01 * \mathrm{H} 2 * \mathrm{D} 2 \wedge 2$ | $=\mathrm{A} 2+(1-\mathrm{H} 2)^{*}(\mathrm{~A} 2-\mathrm{B} 2)^{*}(\mathrm{G} 2)^{\wedge} 2$ | $=\mathrm{B} 2 *(1-\mathrm{H} 2)+0.05^{*}(\mathrm{G} 2)^{\wedge} 2$ | $=(\mathrm{E} 2 / \mathrm{F} 2) * \mathrm{~K} 2+(\mathrm{D} 2 / \mathrm{F} 2) *(1-\mathrm{H} 2) * \mathrm{~L} 2$ |
| 3 | $=\mathrm{C} 3 *(1-\mathrm{H})$ /A3 | $=(\mathrm{C} 3 *(1-\mathrm{H} 3) / \mathrm{A} 3+\mathrm{H} 3 * \mathrm{D} 3-0.01 * \mathrm{H} 3 * \mathrm{D} 3 \wedge 2$ | $=\mathrm{A} 3+(1-\mathrm{H} 3)^{*}(\mathrm{~A} 3-\mathrm{B} 3)^{*}(\mathrm{G} 3)^{2}$ | $=\mathrm{B} 3^{*}(1-\mathrm{H} 3)+0.05^{*}(\mathrm{G} 3)^{\wedge} 2$ | $=(\mathrm{E} 3 / \mathrm{F} 3) * \mathrm{~K} 3+(\mathrm{D} 3 / \mathrm{F}) * *(1-\mathrm{H} 3) *$ L 3 |
| 4 | $=\mathrm{C} 4 *(1-\mathrm{H} 4) / \mathrm{A} 4$ | $=(\mathrm{C} 4 *(1-\mathrm{H} 4) / \mathrm{A} 4+\mathrm{H} 4 * \mathrm{D} 4-0.01 * \mathrm{H} 4 * \mathrm{D} 4 \wedge 2$ | $=\mathrm{A} 4+(1-\mathrm{H} 4)^{*}(\mathrm{~A} 4-\mathrm{B} 4)^{*}(\mathrm{G} 4)^{\wedge} 2$ | $=\mathrm{B4} *(1-\mathrm{H} 4)+0.05^{*}(\mathrm{G} 4)^{\wedge} 2$ | $=($ E4/F4)*K4+(D4/F4)**(1-H4)*L4 |
| 5 | $=\mathrm{C} 5 *(1-\mathrm{H5}) / \mathrm{A} 5$ | $=\left(\right.$ C5* $\left.{ }^{(1-H 5)}\right) / \mathrm{A} 5+\mathrm{H} 5 * \mathrm{D} 5-0.01^{*} \mathrm{H} 5 * \mathrm{D} 5 \wedge 2$ | $=\mathrm{A} 5+(1-\mathrm{H} 5)^{*}(\mathrm{~A} 5-\mathrm{B} 5)^{*}(\mathrm{G} 5)^{2}$ | $=\mathrm{B} 5^{*}(1-\mathrm{H} 5)+0.05^{*}(\mathrm{G} 5)^{\wedge} 2$ | $=(\mathrm{E} 5 / \mathrm{F} 5) * \mathrm{~K} 5+(\mathrm{D} 5 / \mathrm{F} 5) *(1-\mathrm{H} 5) *$ L5 |
| 6 | $=\mathrm{C} 6 *(1-\mathrm{H})$ / A 6 | $=\left(\mathrm{C} 6 *(1-\mathrm{H} 6) / \mathrm{A} 6+\mathrm{H} 6 * \mathrm{D} 6-0.01 * \mathrm{H} 6 * \mathrm{D}^{\wedge} 2\right.$ | $=\mathrm{A} 6+(1-\mathrm{H} 6)^{*}(\mathrm{~A} 6-\mathrm{B} 6)^{*}(\mathrm{G} 6)^{\wedge} 2$ | $=B 6 * *(1-\mathrm{H} 6)+0.05^{*}(\mathrm{G} 6)^{\wedge} 2$ | $=(\mathrm{E} / \mathrm{F} 6) * \mathrm{~K} 6+(\mathrm{D} 6 / \mathrm{F} 6) *(1-\mathrm{H} 6) * \mathrm{~L} 6$ |
| 7 | $=\mathrm{C} 7 *(1-\mathrm{H} 7) / \mathrm{A} 7$ | $=(\mathrm{C} 7 *(1-\mathrm{H} 7) / \mathrm{A} 7+\mathrm{H} 7 * \mathrm{D} 7-0.01 * \mathrm{H} 7 * \mathrm{D} 7 \wedge 2$ | $=\mathrm{A} 7+(1-\mathrm{H} 7)^{*}(\mathrm{~A} 7-\mathrm{B} 7)^{*}(\mathrm{G} 7)^{\wedge} 2$ | $=B 7 *(1-H 7)+0.05 *(G 7)^{\wedge} 2$ | $=(\mathrm{E} 7 / \mathrm{F} 7) * \mathrm{~K} 7+(\mathrm{D} 7 / \mathrm{F7}) * *(1-\mathrm{H} 7) *$ L7 |
| 8 | $=\mathrm{C} 8 *(1-\mathrm{H}) / \mathrm{A} 8$ | $=(\mathrm{C} 8 *(1-\mathrm{H} 8) / \mathrm{A} 8+\mathrm{H} 8 * \mathrm{D} 8-0.01 * \mathrm{H} 8 * \mathrm{D} 8 \wedge 2$ | $=\mathrm{A} 8+(1-\mathrm{H} 8)^{*}(\mathrm{~A} 8-\mathrm{B} 8)^{*}(\mathrm{G} 8)^{2}$ | $=B 8^{*}(1-\mathrm{H} 8)+0.05^{*}(\mathrm{G} 8)^{\wedge} 2$ | $=(\mathrm{E} / \mathrm{F} 8) * \mathrm{~K} 8+(\mathrm{D} 8 / \mathrm{F} 8) *(1-\mathrm{H} 8) * \mathrm{~L} 8$ |
| 9 | = $\mathrm{C} 9 *$ (1-H9)/A9 | $=($ C $9 *(1-\mathrm{H} 9)$ )/A9+H9*D9-0.01*H9*D9^2 | $=\mathrm{A} 9+(1-\mathrm{H} 9) *$ (A9-B9)* ${ }^{(\mathrm{G} 9)^{\wedge} 2}$ | $=B 9 * *(1-H 9)+0.05^{*}(\mathrm{G} 9)^{\wedge} 2$ |  |
| 10 | $=\mathrm{Cl0}$ *(1-H10)/Al0 |  | $=\mathrm{Al0+(1-H10})^{*}(\mathrm{~A} 10-\mathrm{B} 10)^{*}(\mathrm{G} 10)^{\wedge 2}$ | $=\mathrm{B} 10^{*}(1-\mathrm{H} 10)+0.05^{*}(\mathrm{G} 10)^{\wedge} 2$ | $=(\mathrm{E} 10 / \mathrm{F} 10) * \mathrm{~K} 10+(\mathrm{D} 10 / \mathrm{F} 10) *(1-\mathrm{H} 10) * \mathrm{~L} 10$ |
| 11 | $=$ Cl1*(1-H11)/All | $=($ C11** $1-\mathrm{Hl1)}$ )/A11+H11*D11-0.01*H11*D11^2 | $=\mathrm{Al1}+(1-\mathrm{H} 11)^{*}(\mathrm{Al1}-\mathrm{Bl1})^{*}(\mathrm{G} 11)^{\wedge} 2$ | $=\mathrm{B11*}(1-\mathrm{H} 11)+0.05^{*}(\mathrm{G} 11)^{\wedge} 2$ | $=(\mathrm{E} 11 / \mathrm{Fl1}) * \mathrm{~K} 11+($ (D11/F11)*$(1-\mathrm{H} 11) * \mathrm{~L} 11$ |
| 12 | $=\mathrm{Cl12*}(1-\mathrm{Hl2}) \mathrm{Al2}$ | $=\left(\mathrm{Cl2} 2 *(1-\mathrm{H} 12) / \mathrm{AL} 12+\mathrm{H} 12 * \mathrm{D} 12-0.00{ }^{*} \mathrm{H} 12 *\right.$ D $12 \wedge 2$ | $=\mathrm{Al2}+(1-\mathrm{H} 12)^{*}(\mathrm{Al2} 2 \mathrm{~B} 12)^{*}(\mathrm{G} 12)^{\wedge}$ | $=\mathrm{B} 12 *(1-\mathrm{H} 12)+0.05^{*}(\mathrm{G} 12)^{\wedge} 2$ | $=(\mathrm{E} 12 / \mathrm{F} 12) * \mathrm{~K} 12+(\mathrm{D} 12 / \mathrm{F} 12) *(1-\mathrm{H} 12) * \mathrm{~L} 12$ |
| 13 | $=\mathrm{Cl3} *(1-\mathrm{Hl3}) \mathrm{Al3}$ | $=(\mathrm{Cl3} *(1-\mathrm{H} 13)) / \mathrm{Al3}+\mathrm{H} 13 * \mathrm{D} 13-0.01{ }^{*} \mathrm{H} 13 * \mathrm{D} 13 \wedge 2$ | $=\mathrm{Al3+}+(1-\mathrm{H} 13)^{*}(\mathrm{~A} 13-\mathrm{Bl} 3)^{*}(\mathrm{G} 13)^{\wedge} 2$ | $=\mathrm{B} 13^{*}(1-\mathrm{H} 13)+0.05^{*}(\mathrm{G} 13)^{\wedge} 2$ | $=(\mathrm{E} 13 / \mathrm{Fl} 13) * \mathrm{~K} 13+(\mathrm{D} 13 / \mathrm{Fl} 3) *(1-\mathrm{H} 13) * \mathrm{~L} 13$ |
| 14 | $=\mathrm{Cl4*}(1-\mathrm{Hl4}) / \mathrm{Al4}$ | $=(\mathrm{Cl4*}(1-\mathrm{H14)}) / \mathrm{Al4} 14 \mathrm{H} 14 * \mathrm{D} 14-0.01 * \mathrm{H} 14 * \mathrm{D} 14 \wedge 2$ | $=\mathrm{Al4+(1-H14)*}{ }^{(\mathrm{A} 14-\mathrm{Bl} 14)^{*}(\mathrm{G} 14)^{\wedge} 2}$ | $=\mathrm{B} 14 *(1-\mathrm{H} 14)+0.05^{*}(\mathrm{G} 14)^{\wedge} 2$ | $=(\mathrm{El} 4 / \mathrm{F} 14) * \mathrm{~K} 14+(\mathrm{D} 14 / \mathrm{F} 14) *(1-\mathrm{H} 14) * \mathrm{~L} 14$ |

Notes:

1. We assume that the firm $X Y Z$ has a required return on equity (r0) is $20 \%$ when the firm is all equity (no debt), and the firm has a constant borrowing rate (rD) of $5 \%$. Additionally, we assume that XYZ has a perpetual EBIT of $\$ 20$.
2. The levered firm value is generated by $V_{L}=V_{U}+T_{C} D-f(D)$ and $V_{L}=\frac{E B I T\left(1-T_{C}\right)}{r_{0}}+T_{C} D-f(D)$ where
$f(D)=0.01 \mathrm{TD}^{2}$.
3. To generate rE , we used $r_{E}=r_{0}+\left(1-T_{C}\right)\left(r_{0}-r_{D}\right)\left(\frac{D}{E}\right)^{2}$.
4. The tax rate (T) is $40 \%$ and the cost of debt after tax is $r_{D}^{* *}=\left(1-T_{C}\right) a+b\left(\frac{D}{E}\right)^{2}$ and applied as $\mathrm{rD}^{*}=(1-$ 40)(.05)+(.05)(D/E) ${ }^{2}$.
5. In Section 1, E is $\$ 100$ with no corporate taxes assumption and $\mathrm{D}=\$ 0$ as a start. Thus, the levered and unlevered firm values are equal; $\mathrm{VL}=\mathrm{VU}=\mathrm{EBIT} / \mathrm{r} 0=20 / 20=\$ 100$. In Sections 2 and 3 with corporate taxes assumption and when $\mathrm{D}=\$ 0$, E becomes $\$ 60$ since $\mathrm{VL}=[(1-\mathrm{Tc}) \mathrm{EBIT}] / \mathrm{r} 0+\mathrm{TcD}$ and $\mathrm{VL}=[(1-40) 20] / .20+.40(0)=\$ 60$ due to $\mathrm{V}=\mathrm{E}+\mathrm{D}$.

Figure 1: Graphs Corresponding to Three Scenarios

| PROPOSITION I AND II WITH NO TAX | PROPOSITION I AND II WITH TAX | REALISTIC CASE |
| :---: | :---: | :---: |
| FIRM VALUE: PROPOSITION I WITH NO TAXES | M\&M PROPOSITION I: WITH CORPORATE TAXES | REALISTIC CASE: FIRM VALUE |
| COST OF CAPITAL: M\&M PROPOSITION II WITH NO TAXES | M\&M PROPOSITION II: WITH CORPORATE TAXES | REALISTIC CASE: COST OF CAPITAL |

