Target date immunization using duration: An Excel assignment

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ABSTRACT

Target date immunization is a key strategy in using duration to hedge risk in the funding of future liabilities of a bond portfolio. The goal of target date immunization is to make sure the value of the portfolio’s assets is able to cover the cost of its liabilities at a specific point in time regardless of whether interest rates rise or fall. Thus, students are asked to construct an interactive Excel model that shows the effects of interest rate changes on the value of a portfolio where: (1) the duration of assets and liabilities are matched; and (2) where the durations are not matched. The purpose of an Excel-based assignment is to promote constructive learning. A “constructivist” learning approach allows students to acquire and construct knowledge by giving them a mechanism to be part of the problem-solving process. Another reason for using an Excel-based exercise is to help students become more proficient in Excel, a skill now required by many potential employers. Overall, Excel is an excellent tool to provide a more interactive and constructivist learning experience in the classroom.

Keywords: Target Date Immunization, Duration, Finance Education, Computer Modeling, Microsoft Excel

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INTRODUCTION

Bodie, Kane, and Marcus (Investments, 2018, p. 501) lists three reasons why Macaulay (1938) duration is a key concept in managing a bond portfolio. First, duration provides a measure of the average or effective maturity of a portfolio’s cash flows. Second, duration provides a measure of interest rate sensitivity of a bond portfolio. Third, duration is an essential tool in immunizing a portfolio from interest rate risk.

This paper presents an Excel-based assignment that emphasizes the third key concept listed above by allowing students to observe the effects of interest rate price risk (price sensitivity) and reinvestment risk on a bond portfolio. In this exercise, the student examines how duration can be used to immunize or hedge the value of a bond portfolio against changes in interest rates over a specific time period. This is often referred to as a “target date immunization” strategy. The hedge is created by taking advantage of the offsetting nature of bond price sensitivity and returns on reinvestment of interest payments. For example, an increase in interest rates will lead to a decline in bond prices, and a decrease in the value of the bond portfolio. However, this decline in wealth can be offset by reinvesting the interest payments of the bonds at the new higher interest rate.

The goal of hedging or immunization is to have the value of the portfolio’s assets cover the cost of its liabilities at a specific point in time regardless of whether interest rates rise or fall. Thus, in the assignment students will find that immunization occurs when the duration of the assets matches the duration of the liabilities. Students will also discover if the duration of assets and liabilities are not the same, then there will be a mismatch of the ending values. This will result in an increase in variability, and in a possible shortfall in funds to cover the liability.

Another goal of the assignment is to introduce the student to different Excel functions and more complicated formula construction. This should result in students becoming more professional and proficient in their use of Excel, which is a highly desired tech-skill now required by many potential employers (Angeles, 2014). In addition, the use of tables and graphs allow students to “see” how the relationship between bond price changes and reinvestment of interest payments result in a target date immunized portfolio.

Given some of the intermediate techniques used in Excel, this exercise is appropriate for both upper-class undergraduate and graduate finance students. Professors will find this exercise a practical way to introduce their students to both duration and portfolio immunization in undergraduate courses such as Intermediate Finance, Investments, Financial Institutions, and in graduate MBA classes such as Financial Management. Overall, this Excel assignment will provide students a more interactive and constructivist learning experience in the classroom.

MOTIVATION

Professors are always seeking better ways to enhance their students’ learning experience. Research has shown that better learning takes place when both educator and student are more engaged in the process (Rovai, 2004; Tynjaly, 1998). In finance, professors often find themselves writing a series of equations on the board and then mechanically working through several examples. However, many times this results in the underlying lesson or reasoning being lost. One way to boost student understanding of lecture material is to enhance the lectures with Excel spreadsheet assignments.

So, why Excel? Barreto (2015) suggests that:
“A spreadsheet enhances learning for one simple reason: Neuroscience research has shown that we do not pay as much attention to things we find boring…It may seem counterintuitive, but the more you bombard the student with a variety of sensory information, the more they learn.”

Such an approach allows students to take what they are hearing in the lectures and apply it to a constructive outcome. A “constructivist” learning approach is a method that allows students to acquire and construct knowledge by giving them a mechanism to be part of the process in problem-solving (Boethal & Dimock, 1999; Maddux, Johnson, Willis, 1999). Tynjala (1998) finds that students who are exposed to a constructive learning process versus a traditional classroom approach experience higher levels of learning. Thus, Excel can serve as an excellent method to apply constructive learning in the classroom.

Excel has also been associated with an increase in logical thinking skills, which is also related to writing error-free spreadsheets. (Kruck, Maher, & Barkhi, 2003). In addition, Garrett (2015) states it is very important that students learn a more professional approach to Excel programming that targets a deeper understanding of the task at hand rather than learning superficial features or functions, and button clicking. For example, an optimization spreadsheet requires students to employ creative thinking in order to construct formulas that use variables that are directly affected by other factors in the system, i.e., endogenous variables; versus variables outside the system that are unaffected by factors in the system, i.e., exogenous variables (Barreto, 2015). Such an exercise can produce a much more meaningful atmosphere for learning than what can be provided by simply writing an equation on the board during a lecture. In general, it has been shown that students who have prior experience using Excel-based assignments tend to have a better understanding of lectures and complicated formulas. Students have also demonstrated more confidence and empowerment after completing Excel assignments.

In addition, Hess (2005) suggests that not only does Excel spreadsheet analysis enhance understanding of the subject matter, but it also enhances a student’s employability. In most companies today, Excel is a necessity in financial reporting and forecasting. So, it is not surprising that companies expect workers to know how to use Excel and other basic digital skills (Weber, 2015). In a survey of Tuck Business School Alumni, Madahar (2011) finds that 77% of the respondents believe spreadsheets are very important to critical in their job. In fact, Willis (2016) suggests that technology has become such an important part of business, that in order to be successful in their profession, students need to be exposed to a variety of Excel functions and capabilities. However, Ragland and Ramachandran (2014) find there is a “gap” between the Excel functions accounting students “perceive” are important versus the Excel functions potential employers “believe” are important. One way to close this gap is to provide students with an array of problems where they use a wide variety of Excel functions (e.g., data tables, pivot tables, macros, Solver, nested IF statements, etc.).

Also, in a recent review of millions of job postings, Burning Glass Technologies reported that 67% of mid-level skilled jobs required Excel proficiency and other basic technology skills. Angeles (2014) noted that Bob Myhal, CEO of NextHire believes that technical skills are no longer just a nice add-on, but an expectation of employers. And further states, that one of the most desired tech skills is proficiency in Excel. Also, Weber (2015) finds that jobs demanding basic digital skills have a 13% higher median pay ($22.66) versus jobs requiring no digital skills ($20.14). However, the lack of Excel knowledge seems to affect students at all levels. For
example, Browning (2013) finds that even extremely qualified graduates who acquire highly coveted jobs on Wall Street often lack basic spreadsheet capabilities.

Thus, it is apparent there is a strong need for universities to incorporate Excel in the classroom based on the demand and value businesses place on job candidates with a proficiency in Excel. Therefore, by expanding a student’s knowledge and capabilities in Excel should improve their placement opportunities in the job market. Therefore, this paper proposes an Excel-based assignment to teach students how to use bond duration to provide a target date immunization of a portfolio.

SPREADSHEET CONSTRUCTION

The standard duration calculation and duration properties are first introduced and demonstrated during a regular lecture. The completion of the following assignment should take place in a computer lab and should take less than two hours of class time to complete. Students are provided with an Excel file template during the computer lab class time. The Excel file associated with this assignment is available for download at the following link: https://drive.google.com/file/d/1RkotjGNHL5m5nvg7MQANwFxFH2-gMSK/view?usp=sharing.

The Excel template contains five questions that are illustrated as Exhibits, and this study typically illustrates the final answer to the questions. However, the Excel file must be completed in predetermined cell locations to facilitate ease of grading. These predetermined cells are highlighted in certain colors to direct students to the correct cell location for answer entry. A video recording with instructions on setting up the Excel macros and other inputs is available at the following link: https://youtu.be/dWSZDbC9oh4.

DURATION CALCULATIONS

Question 1

In Question 1, students are asked to calculate and plot the duration of three different bonds for various terms to maturity. Students are to assume that the prevailing market rate is 8% for three different bonds from 1990, 2001, and 2013. The three types of bonds include one premium bond, one discount bond, and one deep-discount bond. The use of macros is introduced in the paper to teach students how to create an Excel macro to compute the required durations. However, if preferred, the duration calculations shown in Exhibit 4 (Appendix) can also be found by using Excel data tables. Exhibit 1 (Appendix) illustrates the assignment question.

In order for students to calculate the seventy-five required durations, the students are instructed to create an Excel macro using three given tabs in the spreadsheet. Each tab represents the calculations needed for each of the three requested years (1990, 2001, and 2013). The macro is created in order to produce the duration calculations in a quicker fashion. The following description illustrates how to create the macro using Excel tabs provided to the students. The example will focus on creating the macro for the 1990 premium bond.

The student should click the “Developer” tab in Excel and make sure to select the “1990” tab. The student will note that the correct compounding periods, coupon rate, fV, pmt, I, present value, start date, end date, and duration calculation are already completed. One special note regarding the start and end date is that the exact number of days for a year must be used. This number is 365.2422 due to the definition of the “tropical year” according to the National
Aeronautics and Space Administration (NASA). A “tropical year” is 5 hours, 48 minutes, and 46 seconds longer than 365 days. Therefore, the average year is 365.2422 days long. Traditionally this discrepancy is accounted for by having a leap year consisting of 366 days every 4 years. Nonetheless, this adjustment works well with the start date and end date for the calculation of duration. Otherwise, in 50 years, the duration calculations would be off by 12.11 calendar days. Although the duration function is used and is already set up for the student, please see Wann and Lamb (2016) for an example that allows students to create their own duration tables.

Next, set the orange box for N equal to 0 because the assignment requires the calculation of duration in increments of 2 years. The range is from 2 years to 50 years. This will allow students to see an interesting phenomenon related to deep discount bonds. The student will need a 2 to copy and paste special values to add to the N box. Under the Part column, there is already a 2 available to copy and paste. Click on this box before starting to record the macro. See the illustration in Exhibit 2 (Appendix).

Click Record Macro. Name the macro “Duration1990” and assign the shortcut key to the letter “t.” Click “OK.” Make sure that you have clicked on the “2” in cell I5. Copy and paste special values by choosing the “add” option to cell G8. Then, copy the duration value (H13). Click on cell C14 in the “Duration Chart 2 yr incrmt” tab. Next, it is important to click on “Use Relative References” in the Developer tab. Use paste special value to paste the duration value in cell C14. Then click on cell C15. Be sure to click “Use Relative References” again to deactivate this feature. Click back on the “1990” tab and click back on the cell with the number “2” in it. Click “Stop Recording” in the Developer tab. The code that was generated by creating this macro is called “Duration1990” with the shortcut “Ctrl+t”. The Excel macro code is available upon request from the authors.

Question 1.b asks the student to create a plot of each of the three duration series obtained in Question 1.a. Exhibit 3 (Appendix) shows the resulting graph of the duration answers to Question 1.a. Answers to Question 1.a are found in Exhibit 4 (Appendix). Exhibit 3 (Appendix) illustrates the answer to Question 1.c. Students should notice that all of the duration graphs do not look the same. The deep discount bond (coupon = 2.63%) actually has a curve that increases and then decreases. Basically, it can be noted that for deep discount bonds, duration increases with time to maturity for short-term bonds and then decreases with time to maturity for long-term bonds. This property is typically overlooked in most textbooks and is interesting to illustrate to students.

**Question 2**

Next, the assignment asks the student to calculate duration utilizing the Excel “duration” function. The data in Question 2 will be used as a reference point for introducing the concept of target date immunization. Question 2 is found in Exhibit 5 (Appendix). It requires the calculation of duration for a 7%, 5-year bond with a current market yield of 6.5%.

Students are encouraged to set up the computations so that the duration for an annual, semiannual, or quarterly bond can be calculated. They can see an example of how to create this by looking back at the yearly tabs provided for Question 1 that were used to create the duration graph. The greater portion of this template is taken from that in Wann and Lamb (2016). The correct answers to Question 2 are found in Exhibit 6 (Appendix). Note that orange cells require raw data input while yellow cells require cell-referenced computations. All yellow-shaded areas must have formulas that consist of cell references to the hard-coded numerical data.
requirement allows the final answer to dynamically change if the student desires to see the effect of altering the original data assumptions. For example, the “Adjusted data” column is needed for dynamic calculations when bond interest is compounded semi-annually or quarterly. The green numerical cell contains the final answer for this part of the question.

TARGET DATE IMMUNIZATION

Question 3

Question 3 transitions student learning to target date immunization strategies. In Question 3, students must use the bond from Question 2 to immunize a $100,000,000 liability due in exactly 4.3942 years, which is the duration found in Question 2. Students must calculate the face value (par value) of the bond portfolio that should be purchased today to accumulate $100,000,000 after 4.3942 years. The students must then prove that this portfolio is immunized by taking the steps listed in Exhibit 7 (Appendix). The contents of Question 3 are found in Exhibit 7 (Appendix).

In Question 3.a, the current value of the bond (P) in today’s dollars will equal:

\[ P = \frac{V}{(1+i)^T} = \frac{100,000,000}{(1+0.065)^{4.3942}} = 75,826,307 \]  

(1)

In Equation (1), V equals the needed future value of the bond, i equals market interest rate, and T is equal to the investment horizon, which equals the duration of 4.3942. The answer to Question 3.a is $75,826,307. The bond price found in Question 2 is 102.0778\% of the face value. Therefore, in Question 3.b, the face value (par value) of the bond purchased today should equal:

\[ \text{Face Value} = \frac{P \times \text{Current Bond Price}}{75,826,307} = 74,282,829 \]  

(2)

The answer to Question 3.b is $74,282,829. In Question 3.c, the accumulated value of the coupon payments must be calculated after 4 years. Four years must be used since the actual duration is 4.3942. One must round down to four years and leave the 0.3942 years to be dealt with in Question 3.e. The answer to Question 3.c is $22,916,418 and can be found by computing the future value of the coupon payments:

\[ \text{FVA}_{\text{coupons}} = \text{PMT} \frac{(1+i)^n-1}{i} = 74,282,829 \times 0.07 \frac{(1+0.065)^4-1}{0.065} = 22,916,418 \]  

(3)

The PMT will equal the face value of the purchased bonds times the coupon rate of 7\%. The interest rate is equal to 6.5\% and n is equal to 4 years. The remaining .3942 years will be taken care of in Question 3.e.

In Question 3.d, the student finds the price of the bond after 4 years. Since the bond is a five-year bond, after 4 years, there is only one year left to maturity. Therefore, the price of the bond after 4 years is $74,631,574 and can be found with Equation (4):

\[ P_4 = \frac{\text{Face value} \times \text{coupon rate} + \text{face value}}{1+i} = \frac{74,282,829 \times 0.07 + 74,282,829}{1+0.065} = 74,631,574 \]  

(4)
The answer to Question 3.e. and the final portfolio value at time 4.3942 years can be calculated as:

\[ V_{4.3942} = (P_4 + FVA_{\text{coupons}})(1 + i)^{(D-4)} = (74,282,829 + 22,916,418)(1 + 0.065)^{(4.3942-4)} \]

\[ = 100,000,000 \]

\( D \) is equal to the duration of 4.3942 years, and “(D-4)” captures the last 0.3942 years left until the liability is due. The student finds in Question 3.e that the portfolio is perfectly immunized and equals $100,000,000 due to proper immunization. Thus, immunization can be accomplished by creating a bond portfolio with a duration equal to the investment horizon.

**Question 4**

Question 4 instructs students to test whether the portfolio is immunized when the investment horizon and duration match and interest rates suddenly either fall to 6% or rise to 7% immediately after the investment in the original bond from Question 3. The purpose of this question is to illustrate that portfolio immunization will be achieved when the duration matches the investment horizon, regardless of interest rate changes. The students must test whether the portfolio is immunized by working through steps 4.a to 4.f. The contents of Question 4 are found in Exhibit 8 (Appendix).

Since the face value of the bond still equals the answer in Question 3.b or $74,282,829, the student will start by using Equation (3) to test for proper immunization. In Question 4.a, the accumulated value of the coupon payments must be calculated after 4 years by using Equation (3) and a 6% market interest rate. Four years must again be used since the actual duration is 4.3942 and the question assumes duration matching. Students must again round down to four years and leave the 0.3942 years to be dealt with in Question 4.c. The PMT equals the face value of the purchased bonds times the coupon rate of 7%. The interest rate is equal to 6% and \( n \) is equal to 4 years. The answer to Question 4.a is $22,747,120 and can be found by using Equation (3) to compute the future value of the coupon payments:

\[ FVA_{\text{coupons}} = (74,282,829 \times 0.07)^{(1+0.06)^{4}-1} = 22,747,120 \] (3)

In Question 4.b, the student finds the price of the bond after 4 years, and since the bond is a five-year bond there is only one year left to maturity. Therefore, the price of the bond after 4 years is $74,983,610 and can be found with Equation (4):

\[ P_4 = \frac{(74,282,829 \times 0.07) + 74,282,829}{(1+0.06)} = 74,983,610 \] (4)

The final portfolio value in Question 4.c at time 4.3942 years at 6% is calculated using Equation (5):

\[ V_{4.3942} = (74,983,610 + 22,747,120)(1 + 0.06)^{(4.3942-4)} = 100,001,642 \] (5)

In Question 4.c, the student finds that the portfolio is slightly overfunded and equals $100,001,642 due to proper immunization. As previously stated, proper immunization consists of
setting the duration of the bond equal to the investment horizon. Although interest rates dropped to 6% and coupons are reinvested at that lower rate, the capital gains experienced in the bond price makes up for the difference needed to reach $100,000,000.

In Question 4.d, the accumulated value of the coupon payments must also be calculated after 4 years by using Equation (3). However, now it is assumed that market interest rates have risen to 7%. The same face value found in Question 3.b and n equal to four years must again be used. Students must still round down to four years and leave the 0.3942 years to be dealt with in Question 4.e. The PMT equals the face value of the purchased bonds times the coupon rate of 7%. The interest rate is equal to 7% and n is equal to 4 years. The answer to Question 4.e is $23,086,807 and can be found by using Equation (3) to compute the future value of the coupon payments:

\[
FVA_{\text{coupons}} = (74,282,829 \times 0.07) \frac{(1+0.07)^4-1}{0.07} = $23,086,807 \quad (3)
\]

In Question 4.e, the student finds the price of the bond after 4 years, and since the bond is a five-year bond there is only one year left to maturity. Using Equation (4), the price of the bond after 4 years is $74,282,829:

\[
P_4 = \frac{(74,282,829 \times 0.07) + 74,282,829}{(1+0.07)} = $74,282,829 \quad (4)
\]

The final portfolio value in Question 4.c at time 4.3942 years at 7% is calculated using Equation (5):

\[
V_{4.3942} = (74,282,829 + 23,086,807)(1 + 0.07)^{4.3942-4} = $100,001,638 \quad (5)
\]

The student finds in Question 4.f that the portfolio is slightly overfunded and equals $100,001,638 due to proper immunization. Again, proper immunization consists of setting the duration of the bond portfolio equal to the desired investment horizon. Although interest rates rose to 7% and the bond suffers a slight capital loss, the coupons are reinvested at a higher rate which makes up for the difference needed to reach $100,000,000.

**Question 5**

Question 5 instructs students to test whether the portfolio is immunized if interest rates suddenly fall to 6% immediately after the investment in a 7% coupon, 5-year bond with a starting yield to maturity of 6.5%. The bond is the same as that in Question 3. However, the difference is that the liability is due in 5 years and markets experience a sudden drop in interest rates to 6%. The purpose of this question is to illustrate that portfolio immunization cannot be achieved when the duration of the bond portfolio does not match the investment horizon. The students must test whether the portfolio is immunized by working through steps 5.a through 5.e. The contents of Question 5 are found in Exhibit 9 (Appendix).

The answer to Question 5.a can be found by using Equation (1). The current value of the bond will different from the answer to Question 3.a since the liability is due in 5 years not 4.3942 years. The student finds that the current value of the bond in today’s dollars equals $72,988,084:
In Question 5.b, the student finds that the face value of the bond purchased today will equal $71,502,379 by using Equation (2):

\[
\text{Face Value} = \frac{72,988,084}{1.020778} = 71,502,379
\]  

(2)

In Question 5.c, the accumulated value of the coupon payments must be calculated after 5 years by using Equation (3). Five years can be used since the actual horizon date is equal to 5 years. The PMT equals the face value of the purchased bonds times the coupon rate of 7%. The interest rate is equal to 6% and \( n \) is equal to 5 years. The answer to Question 5.c is $28,117,153 and can be found by computing the future value of the coupon payments:

\[
\text{FVA}_{\text{coupons}} = (71,502,379 \times 0.07) \left(1 + \frac{0.06}{0.06}\right)^5 - 1 = 28,214,589
\]  

(3)

In Question 5.d, the student finds the price of the bond after 5 years. Since the bond is a five-year bond, after 5 years, the price is equal to the face value (par value) of the bond, i.e., the answer found in Question 5.b. Therefore, the price of the bond after 5 years is $71,502,379. The answer to Question 5.e. and the final portfolio value 5 years is $99,716,968 and can be calculated using Equation (5):

\[
V_5 = (71,502,379 + 28,214,589) = 99,716,968
\]  

(5)

The student finds in Question 5.e that the portfolio is not perfectly immunized and falls short of the needed $100,000,000 because of incorrect immunization. The 4.3942 duration of the bond did not equal the required investment horizon of 5 years. Therefore, at year 5, the portfolio is “underfunded” with a value of only $99,716,968. Thus, an extra $283,032 is needed to be properly immunized. If the problem had assumed a rise in interest rates to 7%, then an “overage” of $285,785 would have occurred. Another important point students should be made aware of is the increase in volatility experienced by the non-immunized bond portfolio vs. the immunized bond portfolio.

**CONCLUSION**

This paper provides educators with an example of target date immunization which is a key strategy in how to use duration to hedge risk in the funding of future liabilities of a bond portfolio. Constructivist learning is utilized through Excel by having students immunize a portfolio by ensuring that the value of the portfolio’s assets covers the cost of the firm’s liabilities at a specific future time. This requires students to construct interactive Excel models that show the effects of interest rate changes on the value of a portfolio. Students learn that the future liability is covered regardless of whether interest rates rise or fall. Students examine cases where the duration of assets and liabilities are matched and where the durations are not matched. During this process, students discover that proper immunization is accomplished by creating a bond portfolio with a duration that matches the investment horizon. Thus, this Excel-based assignment promotes constructive learning where can students acquire and construct knowledge.
by being a part of the problem-solving process. Further, such Excel-based problems may also help students become more employable by becoming proficient in Excel.
REFERENCES


APPENDIX

Exhibit 1. Question 1

1. Calculate the durations for the following 3 bonds in two-year time increments starting at year 2 and ending at year 50: 1) 9.18% coupon bond, 2) 5.46% coupon bond, and 3) 2.63% coupon bond. Assume that the going market rate is 8% to calculate duration.
1.a Create a macro that automatically calculates the durations using the provided spreadsheet template (Seen in Exhibit 2).
1.b Plot the durations obtained in the table.
1.c Do all of the graphs of duration look the same? Describe a new property of duration not given in the text as seen in the graph.

Exhibit 2. Excel Macro Starting Point

Exhibit 3. Answer to Question 1.b: Macaulay Duration Graph
Exhibit 4. Answer to Question 1.a: Calculated Duration Data

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Exhibit 5. Question 2

2. Calculate the duration of a 7%, $1 par bond maturing in 5 years if the yield to maturity is 6.5% and interest is paid annually.
2.a Compute the price of the bond using the template below.

Instructions: If the bond is annual, semi-annual, or quarterly, type in a "1", "2", or "4", respectively, in the "Compounding periods per year" Data cell (G21).
2.b Compute the duration of the bond using the "DURATION" function in Excel.
Exhibit 6. Answer to Question 2: Duration Calculation

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<th>Data:</th>
<th>Adjusted data:</th>
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<td>Annual</td>
</tr>
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<td>Coupon %</td>
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<td>7.00%</td>
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<td>$1.00</td>
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<tr>
<td>PMT</td>
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<td>$0.07</td>
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<td>5</td>
</tr>
<tr>
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<td>6.50%</td>
<td>6.50%</td>
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<tr>
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Exhibit 7. Question 3

3. A company needs to pay $100 million in exactly the number of years that is equal to the duration of the 5 year bond. The current interest rate is 6.5%. Show the funding strategy that should be used with the 7% coupon bond.

3.a Calculate the needed current value of the bond in today's dollars. Construct a portfolio of bonds that will accumulate in value to V at time T at the current market interest rate. Set the Macaulay duration D equal to the targeted date of the liability T. The current value of the portfolio of bonds, denoted by P, must be: P = V/(1+i)^T

3.b What should be the face value of the bond purchased today?
3.c What is the accumulated value of the coupon payments after 4 years?
3.d What is the bond price after 4 years?
3.e Now, calculate the bond price plus the coupon value at the duration-matched time to maturity. Is the portfolio immunized? Why or why not?

Exhibit 8. Question 4

4. Next, show how the portfolio is immunized if there is an immediate one-time change in interest rates to (a) 6%, and (b) 7%.

4.a If interest rates fall to 6% right after the purchase of the bond, what is the accumulated value of the coupon payments after 4 years?
4.b What is the bond price after 4 years?
4.c Now, calculate the bond price plus the coupon value at the duration-matched time to maturity. Is the portfolio immunized? Why or why not?
4.d If interest rates rise to 7% right after the purchase of the bond, what is the accumulated value of the coupon payments after 4 years?
4.e What is the bond price after 4 years?
4.f Now, calculate the bond price plus the coupon value at the duration-matched time to maturity. Is the portfolio immunized? Why or why not?
Exhibit 9, Question 5

5. A company needs to pay $100 million 5 years from now. The current interest rate is 6.5%. The company uses the 7% coupon bond from the previous example to pay the liability in 5 years. Will this bond meet the liability if interest rates fall to 6% immediately after the investment in the 7% coupon bond?

5.a Calculate the needed current value of the bond in today's dollars. Construct a portfolio of bonds that will accumulate in value to V at time T at the current market interest rate. Set the Macaulay duration D equal to the targeted date of the liability of 5 years. The current value of the portfolio of bonds, denoted by P, must be: \[ P = \frac{V}{(1+i)^T} \]

5.b What should be the face value of the bond purchased today?

5.c What is the accumulated value of the coupon payments after 5 years?

5.d What is the bond price after 5 years?

5.e Now, calculate the bond price plus the coupon value at the duration-matched time to maturity. Is the portfolio immunized? Why or why not?