

A roulette simulation case: developing problem-solving, statistical analysis, and Excel skills

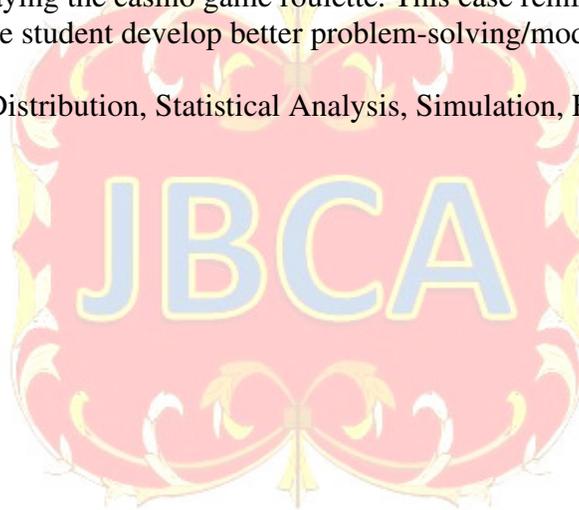
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ABSTRACT

As employers continue to seek business school graduates with problem-solving skills and the ability to use Excel and basic spreadsheet software proficiently, business schools must look for avenues to help students develop these skills. This article provides a case well-suited for an undergraduate or graduate statistics course. In this case, students will use Excel to simulate different strategies for playing the casino game roulette. This case reinforces probability concepts while helping the student develop better problem-solving/modeling skills.

Key Words: Probability Distribution, Statistical Analysis, Simulation, Excel, Problem-Solving



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INTRODUCTION

The competitive environment in which businesses operate makes it critically important that organizations carefully select the new talent they hire. Organizations must use a deliberate process to ensure that new hires possess the necessary job skills to solve problems and make business decisions. Research has identified the skills that employers most desire in employees.

DeBevoise (2018) identified five skills viewed as essential by employers. Among them was "problem-solving—the ability to find solutions to difficult or complex problems." Additionally, five required skills of business school graduates were identified, with proficient use of Excel/spreadsheets included.

A recent survey (The Knowledge, Skills, 2018) of corporate recruiters produced similar findings. Problem-solving was listed as the fourth most required skill of recent business school graduates, with 87% indicating problem-solving as essential. Thirty-one percent of recruiters reported having trouble finding graduates with this skill. Regarding specific skills, advanced knowledge of Excel/spreadsheets ranked fourth, with 74% of employers indicating it is a requirement for effective employment.

Accredited business schools create assurance of learning plans that outline their programs' learning goals and objectives. It is important that colleges and universities prepare students to meet the needs of employers. As such, the skills employers require should be reflected in course learning goals and objectives. Woodside (2020) examined the learning goals of 171 institutions accredited by the American Assembly of Collegiate Schools of Business (AACSB). Approximately 30% of the institutions listed problem-solving and analytical thinking as learning outcomes. This is inconsistent with the requirements stated by employers and suggests that colleges and universities should place more emphasis on these skills.

Business schools can bridge the gap between college graduates' skills and employers' requirements by incorporating more problem-solving, analytical, Excel-based assignments into existing courses. Barr (2013) demonstrated how roulette simulations can be used to teach statistics. Students use a pre-programmed Excel-based simulation as a tutorial to learn statistical concepts. Similarly, Perkins (2021) uses Excel to create a Monte Carlo simulation of a 90-day investment strategy.

This case study uses roulette to teach important statistical concepts while requiring students to reinforce problem-solving and analytical skills. Students also build their Excel skills while creating simulation models.

THE GAME OF ROULETTE

Unlike gambling games such as blackjack, roulette is completely determined by chance. The roulette game is played by dropping a hard ball into a spinning wheel and observing the slot or pocket in which the ball comes to rest. Each slot is given a unique number and is colored RED, BLACK, or GREEN. There are a few versions of the wheel, but the American and European wheels are the most popular (Webb, 2022). The American wheel has 38 slots numbered 00, 0, and 1 through 36. The 00 and 0 slots are colored GREEN. Half of the slots numbered 1 through 36 are colored RED, and the other half are colored BLACK. Figure 1 (Appendix) illustrates the American wheel (Webb, 2022). The only difference in the European wheel is that there is no 00 slot.

Roulette players have quite a few options for predicting where the ball rests after the wheel stops. The player can bet on the color of the slot or whether the number is even or odd. The player can also bet on the winning number falling in any of the following ranges: 1 to 12 (first 12), 13 to 24 (second 12), 25 to 36 (third 12), 1 to 18 (low), and 19 to 36 (high). Additionally, bets may be placed on individual numbers and specific combinations of numbers two, three, four, or more numbers lying next to or near each other on the roulette table. The roulette table layout is shown in Figure 2 (Appendix) (Webb, 2022).

As with any game of chance, the payouts for winning bets reflect the odds of winning on a given spin of the roulette wheel. For example, when betting on RED, the payout is 1-to-1 because there is almost a 50% chance of winning (18 reds out of 38 slots = $18/38 = 0.4737$). A player placing a \$10 bet on RED and winning would receive a \$10 payout when the ball comes to rest in a RED slot. When betting on an individual number, the payout is 35-to-1 because of the low odds of winning (1 winning number out of 38 = $1/38 = 0.0263$). For example, a \$10 bet placed on the number nine pays the player \$350 if the ball rests in slot numbered nine.

The remainder of this paper focuses on building Excel models related to the game of roulette. Three assignments and the corresponding solutions are provided.

CASE ASSIGNMENT 1

Assume a player decides to play the game of roulette using the American Roulette Wheel and Table. The player bets \$10 on RED.

- (1) Create a probability distribution for the random variable X, the net payout on a single spin of the wheel.
- (2) Determine the expected value and standard deviation of the probability distribution.

CASE ASSIGNMENT 1 SOLUTION

When a \$10 bet is placed on RED, there are two possible outcomes: the player loses \$10 or wins \$10. Recall there is a 1-to-1 payout on picking RED as the winning color. Using the classical approach to probability (Lind, 2021), the probability of losing the bet is $20/38$ as 20 of the 38 slots are not RED (18 BLACK and 2 GREEN). The probability of winning the bet is $18/38 = 0.4737$, or slightly less than 50%. The resulting probability distribution for the random variable X, the net payout on a single spin of the wheel, with probabilities rounded to three decimal places, is shown in Figure 3 (Appendix).

The following formulas can be used to find the expected value, $E(X)$, and standard deviation, $\sigma(X)$ of the probability distribution (Lind, 2021):

$$E(X) = \mu = \sum x_i P(x_i)$$

$$\sigma(X) = \sqrt{\sum [(x_i - \mu)^2 * P(x_i)]}$$

The calculations for expected value and standard deviation are shown in Figure 4 (Appendix). The expected value is a loss of \$0.52 and the standard deviation is \$9.99. The expected value of (\$0.52) does not represent the outcome of a single spin of the roulette wheel when betting \$10 on RED since there are only two possible outcomes, lose \$10 or win \$10. Instead, the expected value represents the long-run average payout of betting \$10 on RED over many, many spins of the wheel. Thus, players betting \$10 on RED will average losing \$0.52 per spin of the wheel.

CASE ASSIGNMENT 2

Player's betting \$10 on RED or BLACK can make a profit over the short run of the game, but the negative expected value indicates that a player will lose over the long run. Many betting systems exist to improve the player's chances of winning over the short run. However, all systems have negative expected winnings over the long run. One strategy is the Martingale System (The Martingale, 2022). Using this system, the player places an initial bet, say \$10 on RED. If the player wins the bet, the same amount is wagered on the next spin of the wheel. When the player loses a wheel's spin, the previous bet is doubled for the next spin of the wheel. The player continues to double his bet until he wins a spin. After winning a spin, the player starts the process over by returning to his initial bet. Figure 5 illustrates the betting pattern of the Martingale System (Appendix).

A player with enough money to continually double his bet would always win using the Martingale System. To limit the number of times a player can double his bet, casinos place minimum and maximum bets on individual gaming tables. For example, a table with a minimum bet of \$10 and a maximum bet of \$500 would allow the bettor to double his bet only five times (\$20, \$40, \$80, \$160, \$320).

Using the Martingale System, build an Excel simulation model that allows the user to do the following, as shown in Figure 6 (Appendix):

- (1) Enter the initial bet amount.
- (2) Select RED or BLACK
- (3) Enter the maximum number of doubles.

The model simulates 50 iterations of 1,000 spins when the maximum number of doubles is 3, 4, 5, and 6, calculating the net winnings after each iteration of 1,000 spins. Calculate and interpret summary statistics (mean, median, percentiles, and standard deviation) of the net winnings for 50 iterations at each level of the maximum number of doubles.

CASE ASSIGNMENT 2 SOLUTION

This assignment requires the student to create a lookup table, create a data table, and use the following Excel functions: RAND, LOOKUP, IF, AVERAGE, PERCENTILE, and STDEV.S.

The RAND function is used to simulate random results from the roulette wheel. This function, =RAND(0,1), generates a uniformly distributed number between 0 and 1. Since only the color of the winning number is being considered here, the random numbers must be converted to a color. This is accomplished by using the LOOKUP function, which has the general form =LOOKUP(what,where).

The value that is being looked up (what) is the randomly generated number. It needs to be 'looked up' in a lookup table (where) that is consistent with the probability distribution based only on color, as shown in Figure 7 (Appendix).

Since 2 of the 38 of the slots are GREEN, $2/38=0.0526$ of the range from 0 to 1 must be assigned the color GREEN. Similarly, 18/38 slots are RED, so 18/38 of the range from 0 to 1 must be assigned the color RED. The remaining 18/38 of the range must be assigned to the color BLACK. This mapping of random numbers to the different colors is illustrated in Figure 8 (Appendix). Thus, any randomly generated number between 0 and 0.0526 will be assigned GREEN. Randomly generated numbers between 0.0526 and 0.5263 ($18/38=0.4737 + 0.0526 = 0.5263$) will be assigned the color RED. Any randomly generated numbers greater than 0.5263 will be assigned the color BLACK. The lookup table needed to accomplish this is shown in Figure 9 (Appendix).

Figure 10 (Appendix) illustrates how the color is determined for the first simulated spin. The function =LOOKUP(C9,\$J\$4:\$K\$6), looks up the value in cell C9 (0.1400) in the lookup table (\$J\$4:\$K\$6). Since the random number, 0.1262, is larger than 0.0526 but less than 0.5263, the function returns RED. The color of each subsequent spin is simulated similarly. Note that the spreadsheet could be simplified by replacing C9 with =RAND() in the LOOKUP function.

To complete the simulation, each of the following needs to be determined:

- (1) The amount bet on each spin.
- (2) Whether each spin results in a win or a loss for the player.
- (3) The player's net winnings after each spin.

Except for the amount bet on the first spin, each of these is determined using the Excel =IF function. The =IF function takes the following form: =IF(comparison, true action, false action). The comparison is evaluated as TRUE or FALSE. The true action is returned as the cell's value if the comparison is true. Otherwise, the false action is returned. For example, suppose cell A4 contains =IF(A1<0, "****", "OK"). The value of cell A4 will be **** if A1 is negative. Otherwise, it will read OK.

Figure 11 (Appendix) provides the formulas needed to complete the simulation. Explanations of each formula are listed below for the reader's reference.

Amount Bet

The bet amount on the first spin is set to the initial amount entered in cell E2. The amount bet on subsequent spins of the wheel is dependent on the number of consecutive losses. Following winning spins (0 consecutive losses), the amount bet is the initial bet. Because the Martingale System is being used, the bet following a loss is double the amount of the previous bet, provided the maximum number of doubles has not been reached. The =IF function with an embedded =IF function (nested IF functions) determines the bet amount. The comparison in the first =IF function is evaluated as TRUE if the previous spin was a win (previous consecutive losses equals 0), and the next bet is the initial bet amount. If the function is evaluated as FALSE, another =IF function is used to determine if the maximum number of doubles has been reached. If the maximum number of doubles has been reached, then the initial bet amount is used for the next spin. Otherwise, the previous bet amount is doubled.

Win?

For this entire column, a simple =IF function compares the color of the spin to the bet color (E3). If the colors match, the player wins, and a 1 is recorded. A 0 is recorded when the player loses.

Consecutive Losses

A winning first spin results in 0 consecutive losses, while a losing first spin results in 1 consecutive loss. As with the amount bet, nested =IF functions are used to determine the number of consecutive losses for all subsequent spins. Following winning bets, the number of consecutive losses is set to 0. Following losing bets, 1 is added to the previous number of consecutive losses unless the maximum number of doubles has been reached. In this case, the number of consecutive doubles is set to 1. The amount bet on the first spin is set to the initial bet in cell E2 by entering '=E2' in cell E9. The =IF function is used in column F to indicate the result of each spin of the wheel, lose (0) or win (1). Column G provides the total number of consecutive losses. It is important to record the consecutive number of losses to prevent the simulation for exceeding the maximum number of doubles.

Net Winnings

For the first spin, net winnings are set to the initial bet amount for winning spins and the negative initial bet amount for losing spins. An =IF statement is used for subsequent spins to adjust the previous net winnings by the spin result.

While it is unnecessary, the net winnings from the 1,000 spins are copied to cell K10. A two-variable data table can be used to simulate 50 iterations of 1,000 spins of the wheel as the maximum number of doubles changes. Figure 12 (Appendix) illustrates how the data table can be formatted. In the upper left corner of the table, cell J13 is the net winnings. The top row of the table, cells K13:N13, provides the different maximum number of doubles. These values will be substituted into cell E4. Hence, E4 is the row input cell. To generate 50 iterations for each maximum number of doubles, the numbers 1 to 50 are entered into cells J14:J63. As these values do not impact the simulations, the column input cell should be an empty cell, as shown in Figure 13 (Appendix).

A few observations are worth noting. The average net winnings are negative for all levels of doubling. As the number of doubles increases, there is greater variability in net winnings, and the average net winnings worsens as the number of doubles increases. When a player follows a \$10 losing bet with six consecutive losing double bets, he loses a total of \$1,270, as shown in Figure 15 (Appendix). Overcoming a \$1,270 loss requires winning 127 consecutive \$10 bets, which carries a very small chance of happening.

CASE ASSIGNMENT 3

Another system designed to improve the player's chance of winning is the 3/2 Roulette System (The 3/2 Roulette, 2022). Using this system, the player places two different bets on a single spin. The bets are placed in a 3-to-2 ratio of the value of the bet, such as \$3/\$2, \$6/\$4, and so on.

The roulette table layout, as shown in Figure 2, illustrates the three columns of numbers on which bets can be placed. The columns are also shown in Figure 16. While the first column is evenly split between RED and BLACK numbers, the other two columns have twice as many of one color. The 3/2 Roulette System pairs RED or BLACK with one of these columns to achieve approximately 68% coverage of the entire wheel. For example, a player betting on RED and column 2 would have a 68.4% ($26/38 = 0.6842$) chance of winning because there are 18 RED numbers on the table and an additional 8 BLACK numbers in column 2. Pairing BLACK with column 3 would produce similar results.

Assume a player decides to place a \$3 bet on RED and a \$2 bet on column 2.

- (1) Construct a probability tree diagram illustrating the random variable X, the net payout on a single spin of the wheel.
- (2) Create a probability distribution for the random variable X and determine its expected value.

CASE ASSIGNMENT 3 SOLUTION

One way of constructing the tree diagram is to start with the bet on RED and then consider the bet on column 2. For both bets, there are two outcomes, the player wins or loses the bet. The probability of winning the bet on RED is $18/38$, and its payout is \$3. The probability of losing the bet on RED is $20/38$ (18 BLACK and 2 GREEN numbers), and its payout is -\$3. These probabilities and payouts are illustrated in the left set of branches in the probability tree diagram shown in Figure 17 (Appendix).

The next set of branches represents the second bet on column 2. The corresponding probabilities of winning the second bet are conditional on the results of the first bet. If the first bet is won, then one of the 18 RED numbers is the winning number. Since column 2 has only 4 RED numbers, the chance of winning the second bet is $4/18$, and the chance of losing is $14/18$. If the first bet is lost, then either a BLACK or GREEN number is the winning number. Since there are 20 numbers that are BLACK or GREEN and column 2 has 8 BLACK numbers, the chance of winning the second bet is $8/20$, and the chance of losing it is $12/20$. Note that winning column bets pay 2-to-1. Thus, a \$2 bet on column 2 has a winning payout of \$4.

The general rule of multiplication can be used for the joint probability of each set of first and second-bet outcomes (Lind, 2021). Hence, the probability of winning both bets is found by

$$\begin{aligned} P(\text{Winning Bet 1 and Winning Bet 2}) &= P(\text{Winning Bet 1}) * P(\text{Winning Bet 2} | \text{Winning Bet 1}) \\ &= (18/38) * (4/18) = 4/38 = 0.1053 \end{aligned}$$

The net payouts and corresponding probabilities for each of the four outcomes are shown in Figure 17 (Appendix). Figure 18 (Appendix) shows the probability distribution and expected value of using the 3/2 Roulette System. Note that two of the four scenarios have a \$1 net payout

(win bet 1/lose bet 2 and lose bet 1/win bet 2). As with the Martingale System, the long-run expected payout is negative. Neither the 3/2 Roulette System nor the Martingale System provides the player with a profitable strategy.

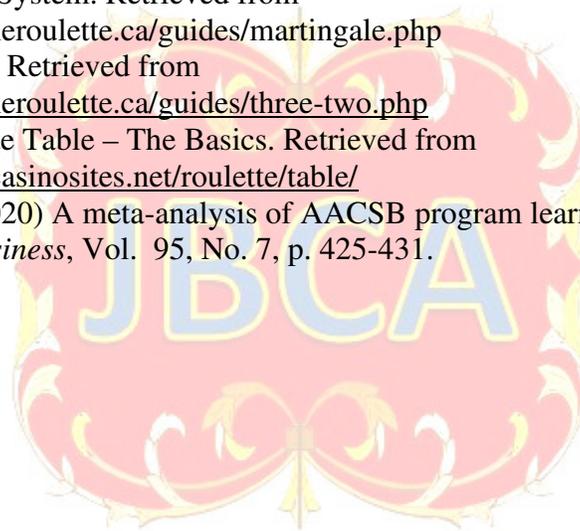
CONCLUSION

To better prepare students to meet the needs of employers, business schools should help students build problem-solving and critical thinking skills using case-based education and learning. Students also need to be proficient in Excel/spreadsheets. This case allows students to build these skills while engaging in a realistic topic that most students and instructors will find entertaining, the roulette gambling game.



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APPENDIX

Figure 1. American Roulette Wheel

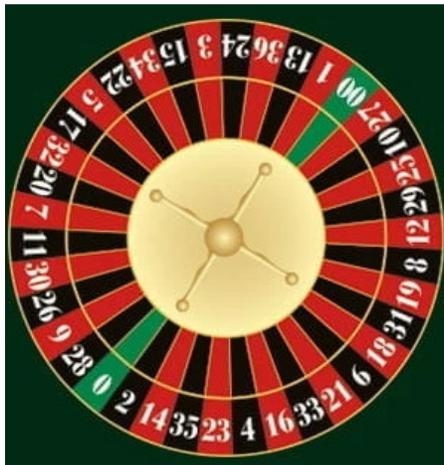


Figure 2. American Roulette Table Layout



Figure 3. Probability Distribution When Betting \$10 on RED

Net Payout, x	P(x)
(\$10)	0.526
\$10	0.474
	1.000

Figure 4. Expected Value and Standard Deviation When Betting \$10 on RED

Net Payout, x	P(x)	x*P(x)	x-mean	(x-mean) ²	(x-mean) ² *P(x)
(\$10)	0.526	(\$5.26)	(\$9.48)	89.8704	47.27
\$10	0.474	\$4.74	\$10.52	110.6704	52.46
Expected Value >>>		(\$0.52)		Variance >>>	99.73
				Standard Deviation >>>	\$9.99

Figure 5. The Martingale System

Initial Bet \$10		
Spin	Result	Next Bet Amount
1	Win	\$10
2	Win	\$10
3	Lose	\$20
4	Lose	\$40
5	Lose	\$80
6	Win	\$10

Figure 6. Simulation Input Data

	A	B	C	D	E
1					
2		Initial Bet			\$10
3		Bet Color (Red/Black)			Red
4		Maximum Number of Doubles			2

Figure 7. Color Probability Distribution

Color	Probability
Green	2/38
Red	18/38
Black	18/38
	38/38

Figure 8. Random Number Mapping



Figure 9. Roulette Lookup Table

J	K
Roulette Lookup Table	
RAND()	Color
0.000000	Green
0.052632	Red
0.526316	Black

Figure 10. Using the LOOKUP Function

D9		=LOOKUP(C9,\$J\$4:\$K\$6)								
A	B	C	D	E	F	G	H	I	J	K
1										
2	Initial Bet			\$10					Roulette Lookup Table	
3	Bet Color (Red/Black)			Red					RAND()	Color
4	Maximum Number of Doubles			2					0.0000	Green
5									0.0526	Red
6									0.5263	Black
7										
8	Roll	Rand	Color	Bet Amount	Win?	Consecutive Losses	Net Winnings			
9	1	0.1820	Red	\$10	1	0	10			
10	2	0.7142	Black	\$10	0	1	\$0			
1007	999	0.2218	Red	\$10	1	0	(\$1,260)		Net Winnings	(\$1,250)
1008	1000	0.5021	Red	\$10	1	0	(\$1,250)			

Figure 11. Spreadsheet Formulas

Column	Row 9	Rows 10-1008
<i>Bet</i>	=E2	=IF(G9=0,\$E\$2,IF(G9>\$E\$4,\$E\$2,2*E9))
<i>Win</i>	=IF(D9=\$E\$3,1,0)	=IF(D10=\$E\$3,1,0)
<i>Consecutive Losses</i>	=IF(F9=1,0,1)	=IF(F10=1,0,IF(G9=\$E\$4+1,1,G9+1))
<i>Net Winnings</i>	=IF(F9=1,E9,-E9)	=IF(F10=1,E10,-E10)+H9

Figure 12. Data Table Layout

	I	J	K	L	M	N
7						
8						
9						
10		Net Winnings	(\$640)			
11						
12						
13		(\$640)	3	4	5	6
14		1				
15		2				
16		3				
17		4				

Figure 13. Data Table Inputs

Data Table ? X

Row input cell: ↑

Column input cell: ↑

Figure 14. Data Table Simulation Results

	I	J	K	L	M	N
9						
10		Net Winnings	(\$150)			
11						
12						
13		(\$150)	3	4	5	6
14		1	(\$340)	(\$2,870)	(\$2,800)	\$970
15		2	(\$630)	(\$2,150)	\$3,530	(\$360)
16		3	(\$1,390)	(\$730)	\$430	(\$6,810)
61		48	(\$110)	(\$450)	(\$400)	\$900
62		49	(\$1,670)	(\$940)	\$470	\$3,350
63		50	(\$1,210)	\$760	\$330	(\$10,800)
64						
65						
66		SUMMARY STATISTICS				
67		Mean	(\$1,013)	(\$1,298)	(\$1,563)	(\$2,539)
68		25th percentile	(\$1,668)	(\$2,188)	(\$2,920)	(\$4,200)
69		Median	(\$1,035)	(\$1,150)	(\$1,845)	(\$2,860)
70		75th percentile	(\$503)	(\$443)	(\$168)	(\$293)
71		90th percentile	\$190	\$467	\$470	\$973
72		Standard Dev.	\$965	\$1,472	\$1,950	\$3,365

Figure 15. Losing Six Doubles

	Bet Amount	Cumulative Loss
Original Bet	\$10	\$10
Double #1	\$20	\$30
Double #2	\$40	\$70
Double #3	\$80	\$150
Double #4	\$160	\$310
Double #5	\$320	\$630
Double #6	\$640	\$1,270

Figure 16. Column Bets

Columns		
First	Second	Third
1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
28	29	30
31	32	33
34	35	36

Figure 17. 3/2 Roulette Probability Tree Diagram

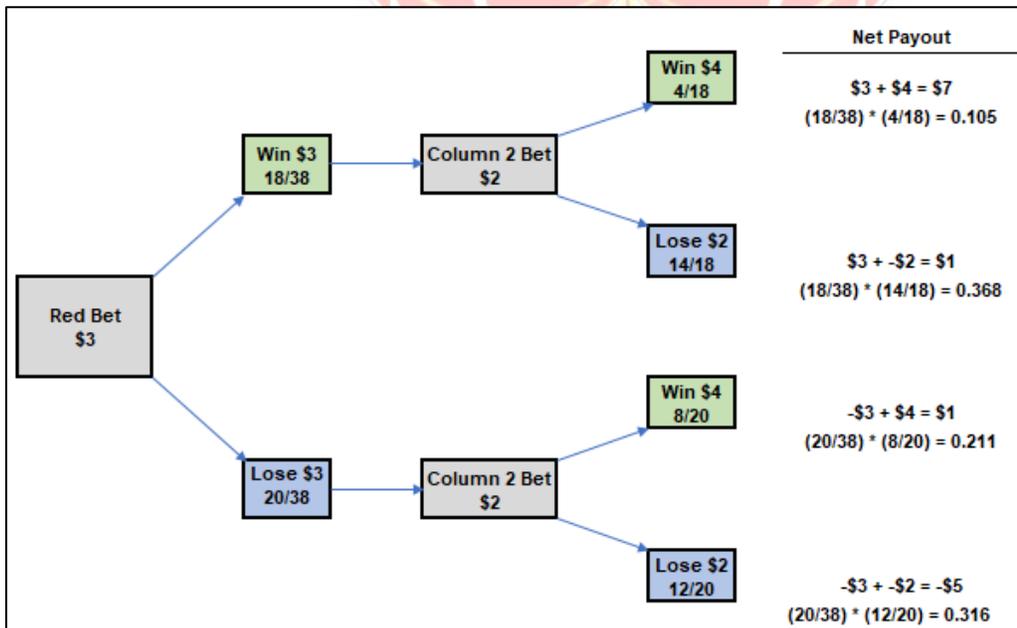


Figure 18. 3/2 Roulette Probability Distribution

Net Payout, x	$P(x)$	$x \cdot P(x)$
(\$5)	0.316	(\$1.58)
\$1	0.579	\$0.58
\$7	0.105	\$0.74
Expected Value >>>		(\$0.27)

