An Economic Decision Model on Mortgage Refinancing

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ABSTRACT

This study provides a straightforward analytical model based on which homeowners can make mortgage refinancing decisions. An illustrative example with sensitivity analysis and decision rules are included to demonstrate the practical application of the model.

Keywords: mortgage refinancing, tax subsidy of interest payments, decision rule

INTRODUCTION

Mortgage refinancing is one of the important decisions to make for many homeowners. Not surprisingly, the literature on mortgage refinancing decisions is extensive and covers various aspects such as timing, behavioral biases, policy implications, and mathematical models.

Campbell (2006) examined various aspects of household financial decision-making, including mortgage refinancing. The research provided evidence on participation, diversification, and mortgage refinancing, suggesting that while many households invest effectively, a minority make significant mistakes. In a related study, Agarwal, Rosen, & Yao (2016) examined the impact of HARP (Home Affordable Refinance Program) on refinancing activity and consumer spending. They found that over three million eligible borrowers refinanced under HARP, saving an average of $3,000 annually, and observed increased durable spending by borrowers after refinancing. In contrast, Frazier & Goodstein (2023) found that "marginal" borrowers (those with low loan balances, low incomes, or low credit scores) were less likely to prepay their mortgage during refinance booms when markets were operating at or near full capacity.

However, arguably the most common reason homeowners refinance is lower interest rate. In this area, there are at least two different representative studies. Distinctly different from the above studies, these two studies collectively provide analytical frameworks for understanding mortgage refinancing decisions, incorporating factors such as borrower heterogeneity, inattention, and market equilibrium effects. Agarwal, Driscoll, & Laibson (2013) derived the first closed-form optimal refinancing rule, providing an exact solution and a second-order approximation for optimal refinancing differentials. In a study with similar motivation, Berger, Olivier, Fuster, & Vavra (2024) built a model of optimal fixed-rate mortgage refinancing incorporating fixed costs and inattention, deriving sufficient new statistics to measure inattention frictions.

The current study is close to this latter line of research stream in that it provided a closed form solution homeowners can use to make refinancing decisions. However, the model used is based on more straightforward algebra than the ones used in the previous two studies.

ANALYTICAL MODEL

To start the discussion, let’s assume the existing mortgage loan has monthly periods of n till maturity, monthly interest rate of I, monthly fixed payment of F, and monthly opportunity cost of y. Then the formula for the sum of present value (PV) of after-tax monthly payments is defined as follows.

Sum of PV (after-tax monthly payments) = PV (total monthly payments) – PV (tax subsidy of interest payments). Since monthly fixed payment, F, for the existing loan is an annuity with n monthly periods, we know that PV (total monthly payments) = F x $\frac{(1-\frac{1}{(1+y)^n})}{y}$

For the present value of tax subsidy on mortgage interest payment, Ro (2020) shows that the PV of tax subsidy for the existing loan is derived as follows.

PV of tax subsidy = $\frac{P_0 IT}{y-I}\left[1-\left(\frac{1+I}{1+y}\right)^n\right]-FT\left[\frac{I}{y(y-I)}-\frac{1}{(1+y)^n}\left[\frac{(1+I)^n}{y-I}+\frac{1}{y}\right]\right]$

where:
P_0 = current principal amount
I = monthly interest rate (i.e., APR/12)

T = personal income tax rate
y = monthly opportunity cost
F = fixed monthly payment
n = number of monthly periods remaining

Putting together, sum of PV (after-tax monthly payments)
= PV (total monthly payments) – PV (tax subsidy)

$$= F \text{ x } \frac{(1-\frac{1}{(1+y)^n})}{y} - \frac{P_0 I T}{y-I}\left[1-\left(\frac{1+I}{1+y}\right)^n\right] + FT\left[\frac{I}{y(y-I)} - \frac{1}{(1+y)^n}\left[\frac{(1+I)^n}{y-I}+\frac{1}{y}\right]\right]$$

This is the closed-form solution for the sum of present value of after-tax monthly payments on the hypothesized existing mortgage loan. This formula considers the changing balance of the loan over time, the difference between the interest rate and the opportunity cost, and provides an accurate calculation of the after-tax payments considering the tax subsidy effect.

Now for a household who is considering refinancing the existing mortgage loan, let's assume that the new loan has the same principal of P_0, the same tax rate of T, monthly periods of m, monthly fixed payment of F', and monthly interest rate I' $(i.e., I' < I)$. Following the same process as before, the sum of present value of after-tax monthly payments can be derived as follows.

First, we start with the formula for the sum of present value of after-tax monthly payments as below.

Sum of PV (after-tax monthly payments) = PV (total monthly payments) – PV (tax subsidy).

Following the same process as with the existing loan, for the new loan,

PV (total monthly payments) = $F' \text{ x } \frac{(1-\frac{1}{(1+y)^m})}{y}$, where F' is the new fixed monthly payment and m is the number of months remaining till maturity.

Second, following Ro (2020), the present value of tax subsidy for the new loan is expressed as follows.

$$\text{PV of tax subsidy} = \frac{P_0 I' T}{y-I'}\left[1-\left(\frac{1+I'}{1+y}\right)^m\right] - F'T\left[\frac{I}{y(y-I')} - \frac{1}{(1+y)^m}\left[\frac{(1+I')^m}{y-I'}+\frac{1}{y}\right]\right]$$

Now putting together, sum of PV (after-tax monthly payments)
= PV (total monthly payments) – PV (tax subsidy)

$$= F' \text{x } \frac{\left(1-\frac{1}{(1+y)^m}\right)}{y} - \frac{P_0 I' T}{y-I'}\left[1-\left(\frac{1+I'}{1+y}\right)^m\right] + F'T\left[\frac{I}{y(y-I')} - \frac{1}{(1+y)^m}\left[\frac{(1+I')^m}{y-I'}+\frac{1}{y}\right]\right]$$

This closed-form solution provides the sum of present value of after-tax monthly payments for the new loan, considering the new loan terms (m, F', and I').

Next, to calculate the financial benefit of refinancing, the present value of after-tax payments in the new loan can be subtracted from the present value of the existing loan. But since there is refinancing cost to get a new loan, the actual financial benefit of refinancing will be (the present value of the existing loan - the present value of after-tax payments in the new loan – refinancing costs). Below, we show the derivation of the financial benefit of refinancing.

For the existing loan:
PV (after-tax monthly payments, existing)

$$= F \text{ x } \frac{\left(1-\frac{1}{(1+y)^n}\right)}{y} - \frac{P_0 I T}{y-I}\left[1-\left(\frac{1+I}{1+y}\right)^n\right] + FT\left[\frac{I}{y(y-I)} - \frac{1}{(1+y)^n}\left[\frac{(1+I)^n}{y-I}+\frac{1}{y}\right]\right]$$

For the new loan:

PV (after-tax monthly payments, new)

$$= F' \text{x } \frac{\left(1-\frac{1}{(1+y)^m}\right)}{y} - \frac{P_0 I' T}{y-I'}\left[1-\left(\frac{1+I'}{1+y}\right)^m\right] + F'T\left[\frac{I}{y(y-I')} - \frac{1}{(1+y)^m}\left[\frac{(1+I')^m}{y-I'}+\frac{1}{y}\right]\right]$$

Therefore, the financial benefit of refinancing in the presence of refinancing cost of K,

= PV (after-tax monthly payments, existing) – PV (after-tax monthly payments, new) – K

$$= F \text{ x } \frac{\left(1-\frac{1}{(1+y)^n}\right)}{y} - \frac{P_0 I T}{y-I}\left[1-\left(\frac{1+I}{1+y}\right)^n\right] + FT\left[\frac{I}{y(y-I)} - \frac{1}{(1+y)^n}\left[\frac{(1+I)^n}{y-I}+\frac{1}{y}\right]\right]$$

$$- \left[F' \text{x } \frac{\left(1-\frac{1}{(1+y)^m}\right)}{y} - \frac{P_0 I' T}{y-I'}\left[1-\left(\frac{1+I'}{1+y}\right)^m\right] + F'T\left[\frac{I}{y(y-I')} - \frac{1}{(1+y)^m}\left[\frac{(1+I')^m}{y-I'}+\frac{1}{y}\right]\right]\right] - K$$

This formula works for all three cases (m>n, m=n, and m<n) because it doesn't make any assumptions about the relative lengths of the loan terms. The values of m and n can be adjusted accordingly for each case:

1. For m>n: Use the actual values of m and n.
2. For m=n: Set m equal to n in the formula.
3. For m<n: Use the actual values of m and n.

The closed-form solution provided above works for all three cases without modification. The difference in loan terms is captured by the different values of m and n, as well as the potentially different monthly interest rates (I and I') and monthly payments (F and F') for the existing and new loans.

DECISION RULES FOR REFINANCING

For the mortgage loan holders (i.e., homeowners) who would like to make refinancing decisions based on the financial benefit of the refinancing, the decision rule for refinancing should be based on whether the financial benefit of the refinancing to be calculated using the above formula is positive or negative:

1. If the financial benefit of the refinancing > 0, then refinancing is beneficial. The homeowner should refinance because the present value of savings from the new loan exceeds the refinancing cost.
2. If the financial benefit of the refinancing = 0, the homeowner is indifferent between refinancing and keeping the existing loan. The savings from refinancing exactly offset the refinancing cost.
3. If the financial benefit of the refinancing < 0, refinancing is not beneficial. The homeowner should keep the existing loan because the present value of savings from the new loan does not exceed the refinancing cost.

It is noteworthy that this decision rule considers the terms of both the existing and new loans (interest rates, monthly payments, and loan periods), the tax implications of mortgage interest deductions, the homeowner's opportunity cost, and the refinancing cost.

By calculating the financial benefit, homeowners can make an informed decision about whether refinancing will provide a net financial benefit, considering both the potential savings in loan payments and the cost of refinancing.

AN ILLUSTRATIVE EXAMPLE

An illustrative example below uses specific numbers to demonstrate the refinancing decision by comparing the PVs of sum of after-tax monthly payments between an existing loan and a new loan, in the presence of a refinancing fee K.

Let's use the following example:

Existing loan:

- Principal balance (P_0): $200,000
- Monthly interest rate (I): 4.3%
- Monthly payment (F): $989.74
- Remaining term (n): 300 months (25 years)

New loan:

- Principal balance (P_0'): $200,000
- Monthly interest rate (I'): 3.5%
- Monthly payment (F'): $898.09
- Term (m): 300 months (25 years)

Common variables:

- Personal income tax rate (T): 10%
- Monthly opportunity cost (y): 0.38% (4.56% annually)
- Refinancing fee (K): $3,000

Step 1: Calculate the present value of sum of after-tax monthly payments for the existing loan

$$\text{PV (existing)} = F \text{ x } \frac{\left(1-\frac{1}{(1+y)^n}\right)}{y} - \frac{P_0 IT}{y-I}\left[1-\left(\frac{1+I}{1+y}\right)^n\right] + FT\left[\frac{I}{y(y-I)} - \frac{1}{(1+y)^n}\left[\frac{(1+I)^n}{y-I} + \frac{1}{y}\right]\right]$$

$$= 989.74 \text{ x } \frac{\left(1-\frac{1}{(1+0.0038)^{300}}\right)}{0.0038} - \frac{200000 \text{ x } 0.00358 \text{ x } 0.1}{0.0038-0.00358}\left[1-\left(\frac{1+0.00358}{1+0.0038}\right)^{300}\right] +$$

$$989.74 \text{ x } 0.1\left[\frac{0.00358}{0.0038(0.0038-0.00358)} - \frac{1}{(1+0.0038)^{300}}\left[\frac{(1+0.00358)^{300}}{0.0038-0.00385} + \frac{1}{0.0038}\right]\right]$$

= $172,373.97

Step 2: Calculate present value of sum of after-tax monthly payments for the new loan

$$\text{PV (new)} = F' \text{x } \frac{\left(1-\frac{1}{(1+y)^m}\right)}{y} - \frac{P_0 I'T}{y-I'}\left[1-\left(\frac{1+I'}{1+y}\right)^m\right] + F'T\left[\frac{I}{y(y-I')} - \frac{1}{(1+y)^m}\left[\frac{(1+I')^m}{y-I'} + \frac{1}{y}\right]\right]$$

$$= 898.09 \text{ x } \frac{\left(1-\frac{1}{(1+0.0038)^{300}}\right)}{0.0038} - \frac{200000 \text{ x } 0.00292 \text{ x } 0.1}{0.0038-0.00292}\left[1-\left(\frac{1+0.00292}{1+0.0038}\right)^{300}\right] +$$

$$898.09 \text{ x } 0.1\left[\frac{0.00292}{0.0038(0.0038-0.00292)} - \frac{1}{(1+0.0038)^{300}}\left[\frac{(1+0.00292)^{300}}{0.0038-0.00292} + \frac{1}{0.0038}\right]\right]$$

= $154,183.57

Step 3: Calculate the financial benefit of refinancing by additionally considering the refinancing fee K.

Financial Benefit of refinancing =

PV (after-tax monthly payment, existing) – PV (after-tax monthly payment, new) - K

= $172,373.97 - $154,183.57 - $3,000

= $15,190.40

Based on the above calculations, it is easy to see how to make refinancing decision: Refinance if the financial benefit > 0. In this case, the financial benefit is $15,190.40, which is greater than zero. Therefore, the homeowner should refinance. The savings from refinancing ($18,190.40) exceed the refinancing fee ($3,000) by $15,190.40, making it a beneficial decision. This example illustrates how to apply the refinancing decision rule by comparing the PVs of sum of after-tax monthly payments between the existing loan and a new loan, while considering the refinancing fee K.

SENSITIVITY ANALYSIS

Since key input variables sometimes change rapidly, it is important for homeowners to understand how sensitive the financial benefit of refinancing when a key input variable changes. To see how sensitive the financial benefit of refinancing is to the change in three important input variables for the new loan (monthly interest rate, monthly opportunity cost, and refinancing cost) the sensitivity analysis has been performed using the base case from the previous example. The results below were obtained by varying each input variable by ±5%, ±10%, and ±15%.

Base case:

- Existing loan: $200,000, 4.3% APR, 300 months
- New loan: $200,000, 3.5% APR, 300 months
- Tax rate: 10%
- Monthly opportunity cost: 0.38% (4.56% annually)
- Refinancing fee: $3,000
- Financial benefit of the base case: $15,190.40

For each line in the following sensitivity analysis, the first number represents % change in the input variable, the second number represents the changed number for the input variable, and the third number indicates the financial benefit when the input variable in focus changed by the first number when all other input variables stay the same. For example, "-15%: 0.248%, the financial benefit: $20,825.62" just below means that while all the other input variables stay the same, when monthly interest rate to new loan changed by -15% (from 0.292% (base case) to 0.248%), the actual monthly interest rate used was 0.248% and the resulting financial benefit was $20,825.62 (vs. $15,190.40 for the base case).

1. Monthly interest rate (new loan):

Base rate: 0.292% (3.5% APR / 12)
-15%: 0.248%, the financial benefit: $20,825.62
-10%: 0.263%, the financial benefit: $19,005.21
-5%: 0.277%, the financial benefit: $17,131.53
+5%: 0.307%, the financial benefit: $13,196.00
+10%: 0.321%, the financial benefit: $11,148.33
+15%: 0.336%, the financial benefit: $9,047.39
Range: $11,778.23

2. Monthly opportunity cost:

Base rate: 0.38%
-15%: 0.323%, the financial benefit: $16,721.85
-10%: 0.342%, the financial benefit: $16,214.70
-5%: 0.361%, the financial benefit: $15,697.55

+5%: 0.399%, the financial benefit: $14,673.25
+10%: 0.418%, the financial benefit: $14,146.10
+15%: 0.437%, the financial benefit: $13,608.95
Range: $3,112.90

3. Refinancing cost:
Base cost: $3,000
-15%: $2,550, the financial benefit: $15,640.40
-10%: $2,700, the financial benefit: $15,490.40
-5%: $2,850, the financial benefit: $15,340.40
+5%: $3,150, the financial benefit: $15,040.40
+10%: $3,300, the financial benefit: $14,890.40
+15%: $3,450, the financial benefit: $14,740.40
Range: $900.00

The input variable that seems most important in determining the financial benefit is the monthly interest rate of the new loan. This can be concluded from the following observation.

First, the widest range of the financial benefit values ($11,778.23) compared to other variables.

Second, the steepest slope when graphed (not reported to avoid the clutter), indicating the highest sensitivity to changes.

Third, the largest absolute changes in the financial benefit values for given percentage changes in the input variable.

Therefore, we can conclude that the monthly interest rate of the new loan has the most significant impact on the refinancing decision, as small changes in this rate can lead to substantial changes in the potential savings from refinancing.

CONCLUSION

In conclusion, this study has provided a straightforward model that could be easily used for evaluating the financial benefits of refinancing. An illustrative example was included to demonstrate the practical application of the model. The findings suggest that homeowners should consider refinancing if the calculated financial benefit is greater than zero. Future research could explore additional factors that may influence refinancing decisions and further refine the model presented in this paper as necessary.

REFERENCES

Agarwal, S., Driscoll, J. C., & Laibson, D. I. (2013). Optimal Mortgage Refinancing: A Closed-Form Solution. Journal of Money, Credit and Banking, 45(4), 591-622.

Agarwal, S., Rosen, R. J., & Yao, V. (2016). Why Do Borrowers Make Mortgage Refinancing Mistakes? Management Science, 62(12), 3494-3509.

Berger, A., Olivier, D., Fuster, A., & Vavra, J. (2024). Optimal Mortgage Refinancing with Inattention. NBER Working Paper, 32447.

Campbell, J. Y. (2006). Household Finance. The Journal of Finance, 61(4), 1553-1604.

Frazier, N., & Goodstein, R. (2023). Is There Crowd Out in Mortgage Refinance? FDIC Center for Financial Research Working Paper No. 2023-01.

Ro, S. J. (2020). How Big is the Tax Subsidy of Home Mortgage Loans?. Southern Business & Economic Journal, *43*(2). 25-37.

Stanton, R. (1995). Rational Prepayment and the Valuation of Mortgage-Backed Securities. The Review of Financial Studies, 8(3), 677-708.